Course Curriculum

Course Title: Heat Transfer II **Course Code: ME 3307**

Week	Topics
1	Physical Mechanism of Convection
2	Classification of Fluid Flows
3	Thermal Boundary Layer
4	Hydraulic boundary layer
5	External Forced Convection + (Quiz 1)
6	Parallel Flow over Flat Plates + (HW 1)
7	Flow across Cylinders and Spheres
8	Flow across Tube Banks
9	Internal Forced Convection + (Exam 1)
10	Laminar Flow in Tubes
11	The Entrance Region
12	Turbulent Flow in Tubes + (Quiz 2)
13	Natural Convection from Finned Surfaces + (HW 2)
14	Natural Convection inside Enclosures
15	Combined Natural and Forced Convection + (Exam 2)
Second Semester Exam	

Heat Transfer-II
Convection heat transfer

Course Description

Heat Transfer II is a required module for mechanical engineering students. Forced convection heat transfer is studied in both internal and external geometries under laminar and turbulent flow regimes. Free convection is also considered where heat transfer is due to flow induced by fluid buoyancy.

Course Topics

1. Hydraulic and Thermal Boundary Layers.
2. Classification of fluid flows.
3. External Forced Convection.
4. Flow over flat plate, across cylinder, across tube banks.
5. Internal Forced Convection.
6. Natural Convection inside Enclosures.

Recommended Textbook(s)

1. J. P. Holman, "Heat Transfer", 9th Edition, 2013.

2. Yunus A. Cengel, "Heat Transfer, A Practical Approach", 2nd Edition, 2012.

3. F. P. Incropera & D. P. Dewitt, "Fundamentals of Heat and Mass Transfer", 2011.

Lab Topics

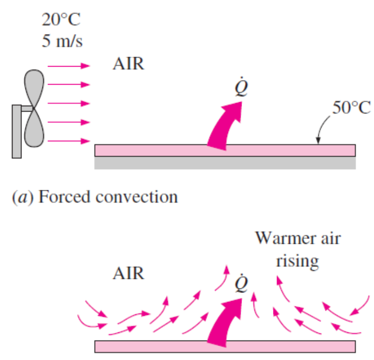
1. Natural Convection Heat Transfer.
2. Forced Convection Heat Transfer
3. Double Pipe Heat Exchangers (effect of inlet fluid temperature).
4. Double Pipe Heat Exchangers (effect of cold fluid mass flowrate).
5. Double Pipe Heat Exchangers (effect of cold fluid flow direction).

Chapter one

Principles of Convection

1.1 Physical Mechanism of Convection

Convection heat transfer is complicated by the fact that it involves fluid motion as well as heat conduction. The fluid motion enhances heat transfer, since it brings hotter and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid. Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction. In fact, the higher the fluid velocity, the higher the rate of heat transfer.



(*a*) Forced convection
(*b*) Free convection

Experience shows that convection heat transfer strongly depends on

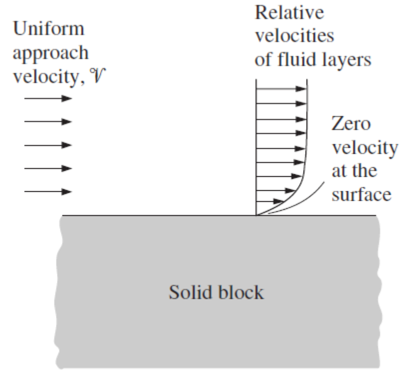
(1) the fluid properties; dynamic viscosity, thermal conductivity k, density, and specific heat C_p,
(2) the geometry and the roughness of the solid surface,
(3) the type of fluid flow (such as being streamlined or turbulent).

The rate of convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by ***Newton's law*** of cooling as

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad \text{(W)} \tag{1.1}$$

The convection heat transfer coefficient (*h*) can be defined as: *the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference*.

When a fluid is forced to flow over a solid surface that is nonporous (i.e., impermeable to the fluid), it is observed that the fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, the fluid layer in direct contact with a solid surface "sticks" to the surface and there is no slip. In fluid flow, this phenomenon is known as the no-slip condition, and it is due to the viscosity of the fluid



The no-slip condition is responsible for the development of the velocity profile for flow. Because of the friction between the fluid layers, the layer that sticks to the wall slows the adjacent fluid layer, which slows the next layer, and so on. A consequence of the no-slip condition is that all velocity profiles must have zero values at the points of contact between a fluid and a solid. *The only exception to the no-slip condition occurs in extremely rarified gases.*

A similar phenomenon occurs for the temperature. When two bodies at different temperatures are brought into contact, heat transfer occurs until both bodies assume the same temperature at the point of contact. Therefore, a fluid and a solid surface will have the same temperature at the point of contact. This is known as *no-temperature-jump condition*. [An implication of the no-slip and the no-temperature jump conditions is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by pure conduction, since the fluid layer is motionless],

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k_{\text{fluid}} \frac{\partial T}{\partial y}\bigg|_{y=0} \qquad (\text{W/m}^2) \tag{1.2}$$

where T represents the temperature distribution in the fluid and $(\partial T/\partial y)_{y=0}$ is the *temperature gradient* at the surface.

1.2 Nusselt Number

It is common practice to nondimensionalize the heat transfer coefficient (h) with the Nusselt number, defined as

$$\text{Nu} = \frac{hL_c}{k} \tag{1.3}$$

where k is the thermal conductivity of the fluid and L_c is the characteristic length.

To understand the physical significance of the Nusselt number, consider a fluid layer of thickness L and temperature difference $\Delta T = T_2 - T_1$. Heat transfer through the fluid layer will be by convection when the fluid involves some motion and by conduction when the fluid layer is motionless. Heat flux (the rate of heat transfer per unit time per unit surface area) in either case will be

$$\dot{q}_{\text{conv}} = h\Delta T$$

$$\dot{q}_{\text{cond}} = k\frac{\Delta T}{L}$$

Taking their ratio gives

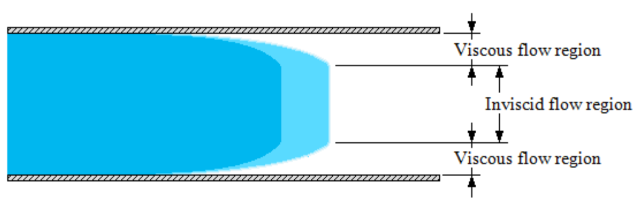
$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu} \qquad (1.4)$$

Therefore, the Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. The larger the Nusselt number, the more effective the convection.

1.3 Classification of Fluid Flows

1.3.1 Viscous versus Inviscid Flow

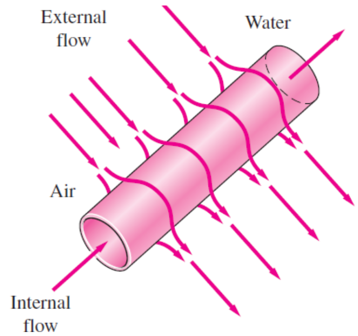
There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the effects of viscosity are significant are called *viscous flows*. The effects of viscosity are very small in some flows, and neglecting those effects greatly simplifies the analysis without much loss in accuracy. Such idealized flows of zero-viscosity fluids are called frictionless or *inviscid flows*.



1.3.2 Internal versus External Flow

A fluid flow is classified as being internal and external, depending on whether the fluid is forced to flow in a confined channel or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is *external flow*. The flow in a pipe or duct is *internal flow* if the fluid is completely bounded by solid surfaces.

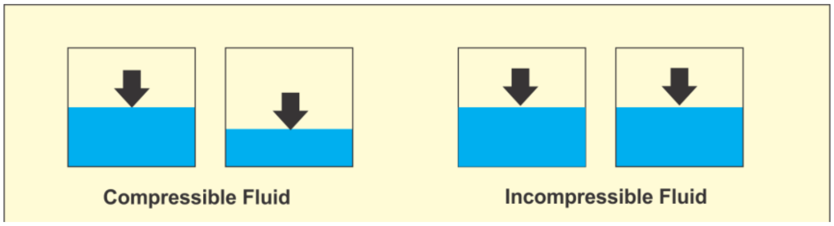
The flow of liquids in a pipe is called open-channel flow if the pipe is partially filled with the liquid and there is a free surface. The flow of water in rivers are examples of such flows.



1.3.3 Compressible versus Incompressible Flow

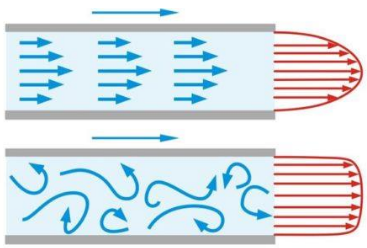
A fluid flow is classified as being compressible or *incompressible*, depending on the density variation of the fluid during flow. The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually classified as incompressible substances.

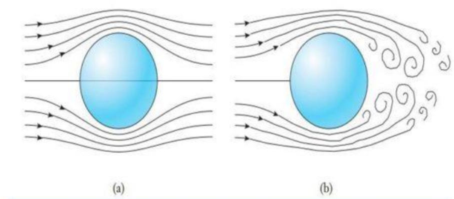
Gases, on the other hand, are highly *compressible*. However, gases flow can be treated as incompressible if the density changes are under about 5%. Note that the flow of a gas is not necessarily a compressible flow.



1.3.4 Laminar versus Turbulent Flow

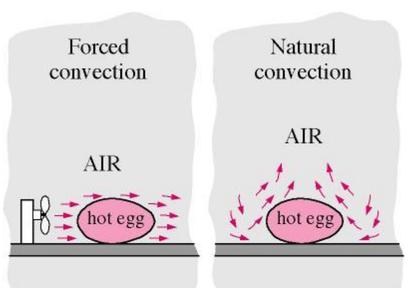
Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth streamlines is called *laminar*. The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities characterized by velocity fluctuations is called *turbulent*. The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the heat transfer rates and the required power for pumping.





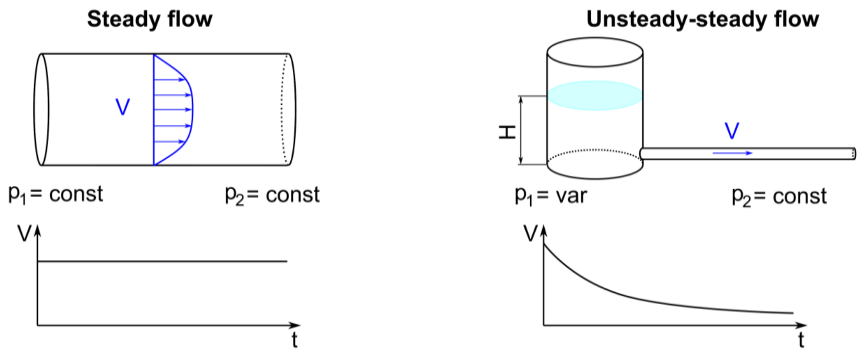
1.3.5 Natural (or Free) versus Forced Flow

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In forced flow, a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural flows, any fluid motion is due to a natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid.



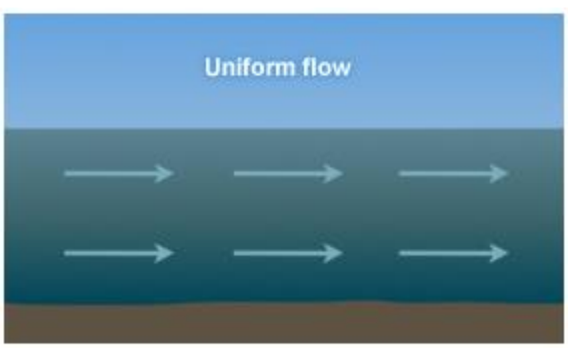
1.3.6 Steady versus Unsteady (Transient) Flow

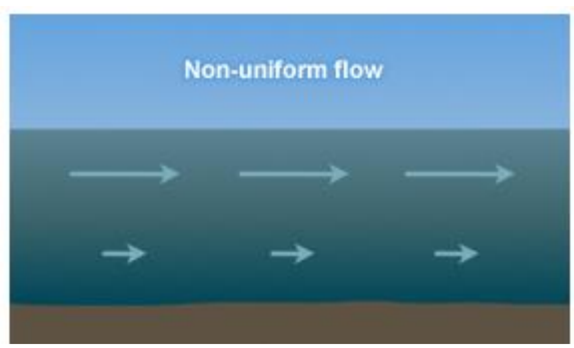
The terms *steady* and *uniform* are used frequently in engineering, and thus it is important to have a clear understanding of their meanings. The term *steady* implies no change with time. The opposite of steady is *unsteady*, or *transient*. The term *uniform*, however, implies no change with location over a specified region. Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as steady-flow devices.



1.3.7 Uniform and non-uniform flow

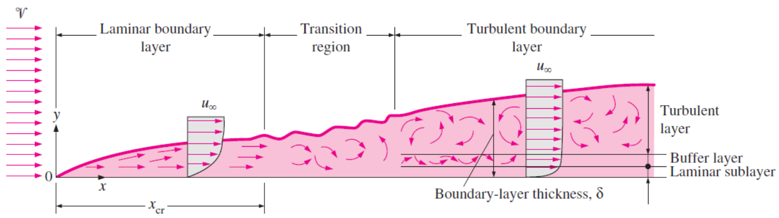
The uniform flow is defined as when the velocity at a given instant of time is same in both magnitude and direction at all points in the flow, the flow is said to be uniform flow. While the non-uniform flow is defined as when the velocity changes from point to point in a flow at any given instant of time, the flow is described as non-uniform flow.



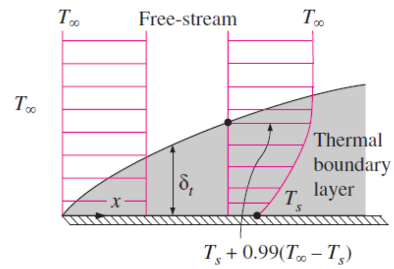


1.4 Thermal Boundary Layer

The velocity boundary layer develops when a fluid flows over a surface. The velocity boundary layer is defined as the region in which the fluid velocity varies from zero to $0.99u_\alpha$. Likewise, the thermal boundary layer develops when a fluid at a specified temperature flows over a surface that is at a different temperature.



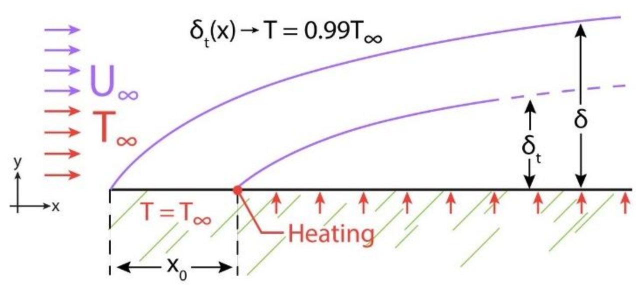
Hydraulic boundary layer



Thermal boundary layer

The thickness of the thermal boundary layer δ_t at any location along the surface is defined as the distance from the surface at which the temperature difference $T-T_s$ equals $0.99(T_\alpha-T_s)$.

Noting that the fluid velocity will have a strong influence on the temperature profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a strong effect on the convection heat transfer.



Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the dimensionless parameter Prandtl number, defined as

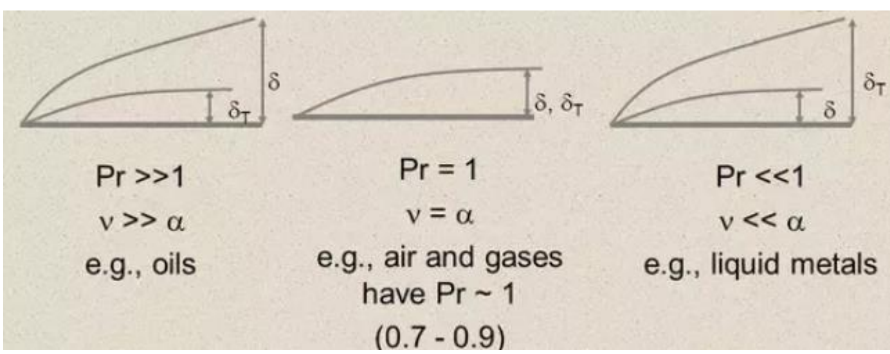
$$Pr = \frac{Momentum\ Diffusivity}{Heat\ Diffusivity} = \frac{\nu}{\alpha} = \frac{(\frac{\mu}{\rho})}{(\frac{k}{\rho c_p})} = \frac{c_p \mu}{k} \quad (1.5)$$

The Pr ≈1 for gases, which indicates that both momentum and heat dissipate through the fluid at about the same rate. Heat diffuses very quickly in liquid metals (Pr <<1) and very slowly in oils (Pr>>1) relative to momentum. Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

TABLE 6–2

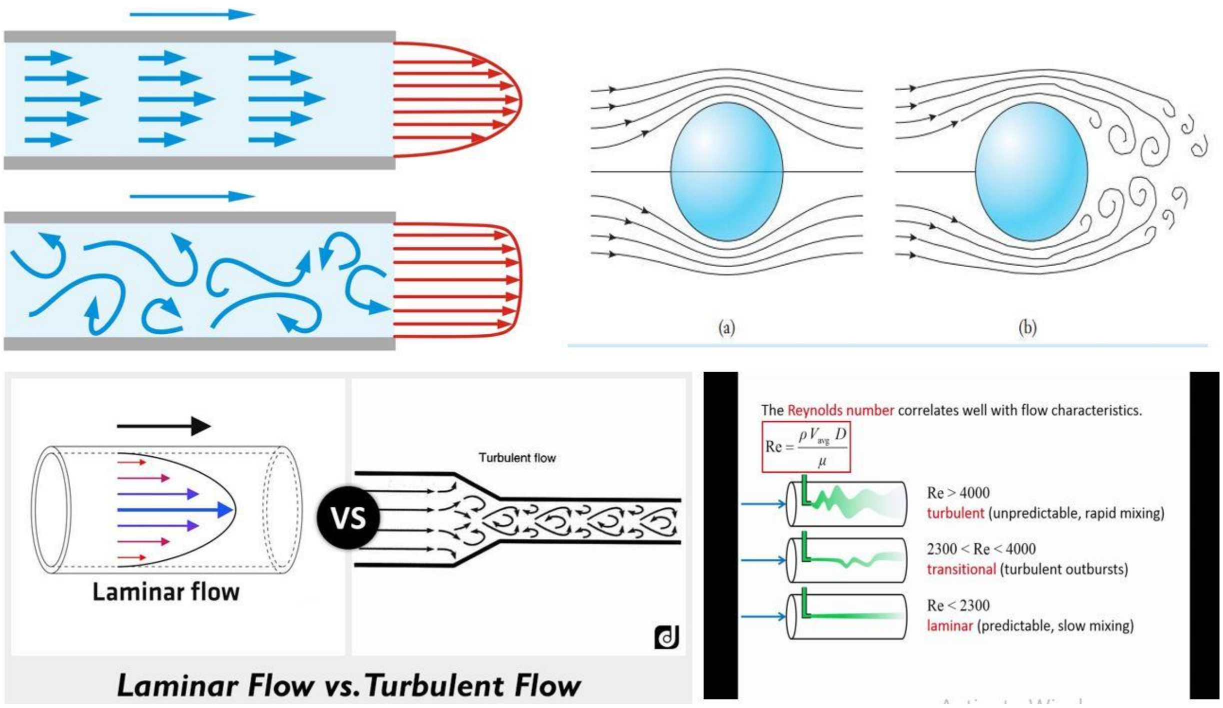
Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000



1.5 Laminar and Turbulent Flows

The flow regime in the first case is said to be laminar, characterized by smooth streamlines and highly-ordered motion, and turbulent where it is characterized by velocity fluctuations and highly-disordered motion. The transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.



The mixing of the fluid in turbulent flow due to the fluctuations could enhance heat and momentum transfer between fluid particles, which increases the friction force on the surface and the convection heat transfer rate. Both the friction and heat transfer coefficients reach maximum values when the flow becomes fully turbulent. The enhancement in heat transfer in turbulent flow does not come for free, however, it may be necessary to use a larger pump to overcome the larger friction forces accompanying the higher heat transfer rate.

Reynolds Number

The transition from laminar to turbulent flow depends on the surface *geometry*, *fluid velocity*, and *fluid properties*, in other words, depends on the ratio of the inertia forces to viscous forces in the fluid. This ratio is called the *Reynolds number*, which is a dimensionless quantity, and is expressed for external flow

$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{\mathcal{V} L_c}{\nu} = \frac{\rho \mathcal{V} L_c}{\mu} \tag{1.6}$$

where L_c is the *characteristic length* of the geometry. For a flat plate, the L_c is the distance x from the leading edge, and is the diameter for the internal flows. The $\nu=\mu/\rho$ (m^2/s) is the *kinematic viscosity* of the fluid. Its unit is identical to the unit of *thermal diffusivity*, and can be viewed as *viscous diffusivity* or *diffusivity for momentum.*

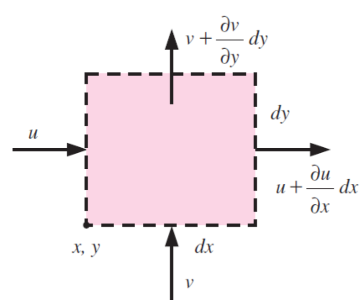
	Laminar	**Transient**	**turbulent**
Internal flow:	$\mathrm{Re} \le 2300$	$2300 \le \mathrm{Re} \le 4000$	$\mathrm{Re} \ge 4000$
External flow:	$\mathrm{Re} < 5\times10^5$	$\mathrm{Re} = 5\times10^5$	$\mathrm{Re} > 5\times10^5$

1.6 Derivation of Differential Convection Equations

1.6.1 Conservation of Mass Equation

$$\begin{pmatrix}\text{Rate of mass flow}\\ \text{into the control volume}\end{pmatrix} = \begin{pmatrix}\text{Rate of mass flow}\\ \text{out of the control volume}\end{pmatrix}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$



This is the conservation of mass relation, also known as the *continuity equation*, or *mass balance* for steady two-dimensional flow of a fluid with constant density.

1.6.2 Conservation of Momentum Equations

The conservation of momentum can be stated as: [the net force acting on the control volume is equal to the mass times the acceleration of the fluid element within the control volume, which is also equal to the net rate of momentum outflow from the control volume].

$$(\text{Mass})\begin{pmatrix}\text{Acceleration}\\ \text{in a specified direction}\end{pmatrix} = \begin{pmatrix}\text{Net force (body and surface)}\\ \text{acting in that direction}\end{pmatrix}$$

For 3D transient and compressible flow;

X-momentum

$$\rho\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+W\frac{\partial U}{\partial z}\right)=-\frac{\partial P}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right)$$

Y-momentum

$$\rho\left(\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+W\frac{\partial V}{\partial z}\right)=-\frac{\partial P}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}\right)$$

Z-momentum

$$\rho\left(\frac{\partial W}{\partial t}+U\frac{\partial W}{\partial x}+V\frac{\partial W}{\partial y}+W\frac{\partial W}{\partial z}\right)=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}\right)$$

1.6.3 Conservation of Energy Equation

$$(\dot{E}_{\text{in}}-\dot{E}_{\text{out}})_{\text{by heat}}+(\dot{E}_{\text{in}}-\dot{E}_{\text{out}})_{\text{by work}}+(\dot{E}_{\text{in}}-\dot{E}_{\text{out}})_{\text{by mass}}=0$$

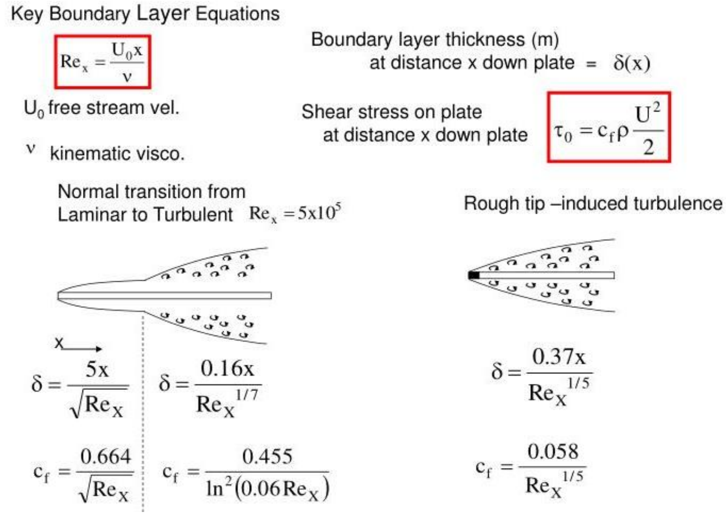
$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

which states that the net energy convected by the fluid out of the control volume is equal to the net energy transferred into the control volume by heat conduction.

1.7 Hydraulic boundary layer thickness (laminar flow)

$$\delta = \frac{5.0}{\sqrt{u_\infty / \nu x}} = \frac{5.0x}{\sqrt{Re_x}}$$

$$\tau_w = 0.332 u_\infty \sqrt{\frac{\rho \mu u_\infty}{x}} = \frac{0.332 \rho u_\infty^2}{\sqrt{Re_x}}$$



For Laminar flow:

$$\delta = \frac{4.91x}{Re_x^{1/2}} \quad \text{and} \quad C_{f,x} = \frac{0.664}{Re_x^{1/2}}, \quad Re_x < 5 \times 10^5 \tag{1.7}$$

$$C_f = \frac{1.33}{Re_L^{1/2}} \qquad Re_L < 5 \times 10^5 \qquad \text{For entire plate} \tag{1.8}$$

For turbulent flow:

$$\delta = \frac{0.38x}{Re_x^{1/5}} \quad \text{and} \quad C_{f,x} = \frac{0.059}{Re_x^{1/5}}, \quad 5 \times 10^5 \le Re_x \le 10^7 \tag{1.9}$$

$$C_f = \frac{0.074}{Re_L^{1/5}} \qquad 5 \times 10^5 \le Re_L \le 10^7 \qquad \text{For entire plate} \tag{1.10}$$

1.8 Thermal boundary layer thickness (laminar flow)

$$\delta_t = \frac{\delta}{Pr^{1/3}} = \frac{5.0x}{Pr^{1/3}\sqrt{Re_x}} \tag{1.11}$$

1.9 Local convection heat transfer coefficient and Nusselt number (laminar flow)

$$h_x = \frac{q_s}{T_s - T_\infty} = \frac{-k(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} = 0.332\ \mathrm{Pr}^{1/3} k \sqrt{\frac{u_\infty}{\nu x}} \tag{1.12}$$

$$h = \frac{1}{A_s}\int_{A_s} h_{\mathrm{local}} dA_s \quad \text{and} \quad h = \frac{1}{L}\int_0^L h_x dx \tag{1.13}$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\ \mathrm{Pr}^{1/3}\mathrm{Re}_x^{1/2} \qquad \mathrm{Pr} > 0.6 \tag{1.14}$$

$$\mathrm{St} = \frac{h}{\rho C_p \mathcal{V}} = \frac{\mathrm{Nu}}{\mathrm{Re}_L \mathrm{Pr}} \tag{1.15}$$

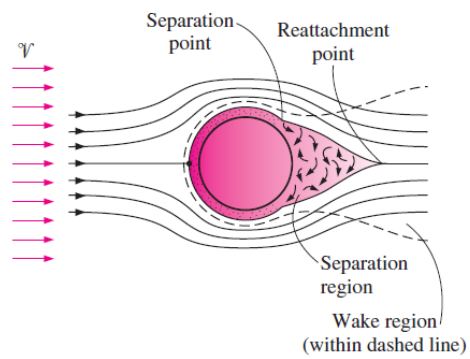
which is the *Stanton number*, which is also a dimensionless heat transfer coefficient.

Chapter two

External Forced Convection

2.1 Drag and Heat Transfer in External Flow

The experimental data for heat transfer is often represented conveniently with reasonable accuracy by a simple power-law relation of the form:



$$Nu = C\,Re_L^m Pr^n \tag{2.1}$$

where m and n are constant exponents, and the value of the constant C depends on geometry and flow.

The fluid properties also vary with temperature, and thus with position across the boundary layer. In order to account for the variation of the properties with temperature, the fluid properties are usually evaluated at the so-called *film temperature*, defined as

$$T_f = \frac{T_s + T_\infty}{2} \tag{2.2}$$

2.2 Parallel Flow over Flat Plates

2.2.1 Friction Coefficient

The *local boundary layer thickness* and the *local friction coefficient* at location x for laminar flow over a flat plate

Laminar: $\delta_{v,x} = \dfrac{5x}{Re_x^{1/2}}$ and $C_{f,x} = \dfrac{0.664}{Re_x^{1/2}}, \quad Re_x < 5 \times 10^5$ (2.3)

The corresponding relations for turbulent flow are

Turbulent: $\delta_{v,x} = \dfrac{0.382x}{Re_x^{1/5}}$ and $C_{f,x} = \dfrac{0.0592}{Re_x^{1/5}}, \quad 5 \times 10^5 \le Re_x \le 10^7$ (2.4)

The local friction coefficients are higher in *turbulent* flow than they are in *laminar* flow because of the intense mixing that occurs in the turbulent boundary layer.

The *average friction coefficient* over the entire plate is determined by

Laminar: $C_f = \dfrac{1.328}{Re_L^{1/2}} \qquad Re_L < 5 \times 10^5$ (2.5)

Turbulent: $C_f = \dfrac{0.074}{Re_L^{1/5}} \qquad 5 \times 10^5 \le Re_L \le 10^7$ (2.6)

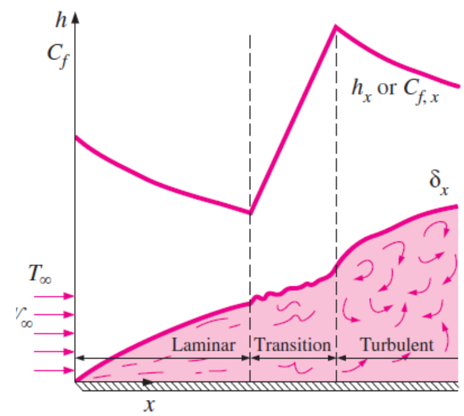
The friction coefficient for the turbulent flow over a flat plate with surface roughness is

Rough surface, turbulent: $C_f = \left(1.89 - 1.62 \log \dfrac{\varepsilon}{L}\right)^{-2.5}$ (2.6)

where ε refers to the roughness height (m).

2.2.2 Heat Transfer Coefficient

The *local Nusselt number* at a location x for laminar flow over a flat plate was determined in Chapter 1 by solving the differential energy equation to be



Laminar: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.60 \tag{2.7}$$

The corresponding relation for turbulent flow is

Turbulent: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5\times10^5 \le \mathrm{Re}_x \le 10^7 \end{array} \tag{2.8}$$

The *average Nusselt number* over the entire plate is determined by

Laminar: $$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re}_L < 5\times10^5 \tag{2.9}$$

Turbulent: $$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5\times10^5 \le \mathrm{Re}_L \le 10^7 \end{array} \tag{2.10}$$

Or

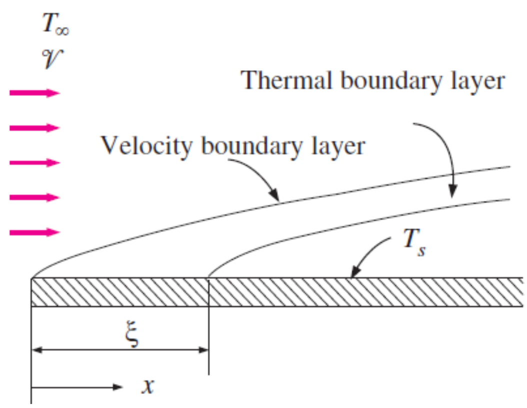
$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5\times10^5 \le \mathrm{Re}_L \le 10^7 \end{array} \tag{2.11}$$

It is desirable to have a single correlation that applies to all fluids, including liquid metals. By curve-fitting existing data, the following correlation was proposed to be applicable for all Prandtl numbers and for smooth surface and is claimed to be accurate to ±1%,

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\ \mathrm{Pr}^{1/3}\ \mathrm{Re}_x^{1/2}}{[1 + (0.0468/\mathrm{Pr})^{2/3}]^{1/4}} \tag{2.13}$$

2.3 Flat Plate with Unheated Starting Length

Consider a flat plate whose heated section is maintained at a constant temperature ($T = T_s = constant$) for $x > \xi$. The local Nusselt numbers for both laminar and turbulent flows are determined to be



Laminar:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\ \mathrm{Re}_x^{0.5}\ \mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}} \tag{2.14}$$

Turbulent:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\ \mathrm{Re}_x^{0.8}\ \mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}} \tag{2.15}$$

Note $x > \xi$.

Laminar:
$$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L} h_{x=L} \tag{2.16}$$

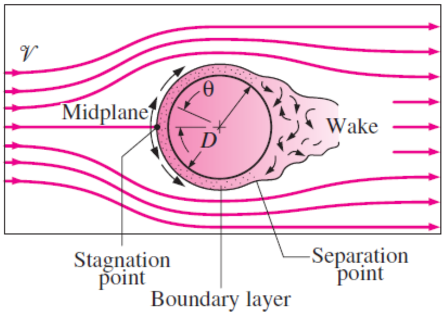
Turbulent:
$$h = \frac{5[1-(\xi/x)^{9/10}]}{4(1-\xi/L)} h_{x=L} \tag{2.17}$$

When a flat plate is subjected to *uniform heat flux* instead of uniform temperature, the local Nusselt number is given by

Laminar: $$\mathrm{Nu}_x = 0.453\ \mathrm{Re}_x^{0.5}\ \mathrm{Pr}^{1/3} \quad (2.18)$$

Turbulent: $$\mathrm{Nu}_x = 0.0308\ \mathrm{Re}_x^{0.8}\ \mathrm{Pr}^{1/3} \quad (2.19)$$

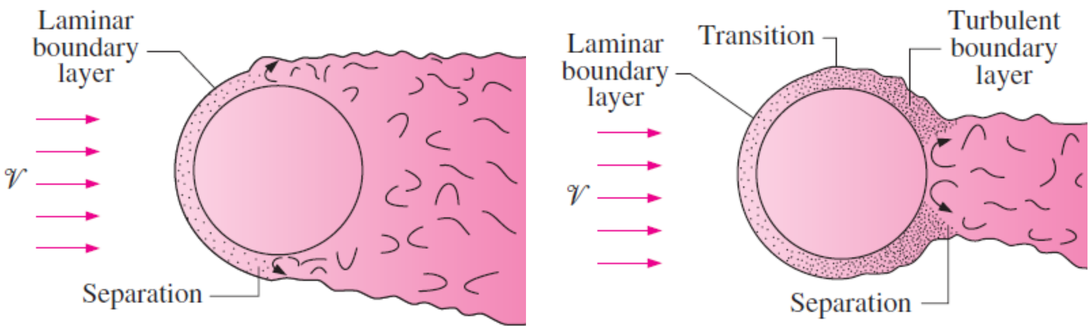
2.4 Flow across Cylinders and Spheres



Flow across cylinders and spheres is frequently encountered in practice. For example, the tubes in a shell-and-tube heat exchanger involve both internal flow through the tubes and external flow over the tubes, and both flows must be considered in the analysis of the heat exchanger. Also, many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be *the external diameter D*. Thus, the Reynolds number is defined as Re= VD/ν where V is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $Re_{cr} = 2\times10^5$. That is, the boundary layer remains laminar for about $Re \le 2\times10^5$ and becomes turbulent for $Re \ge 2\times10^5$.

Flow separation occurs at about θ=80° (measured from the stagnation point) when the boundary layer is laminar and at about θ=140° when it is turbulent.



For flat-plate:

$$C_D = C_{D,\ \text{friction}} = C_f$$

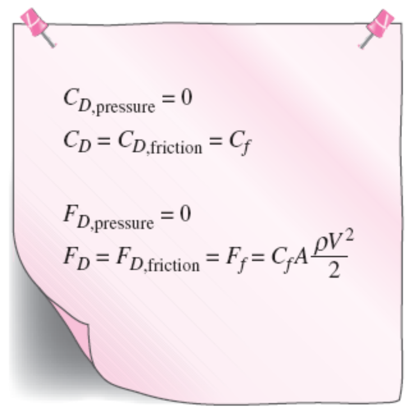


FIGURE 7–4
For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

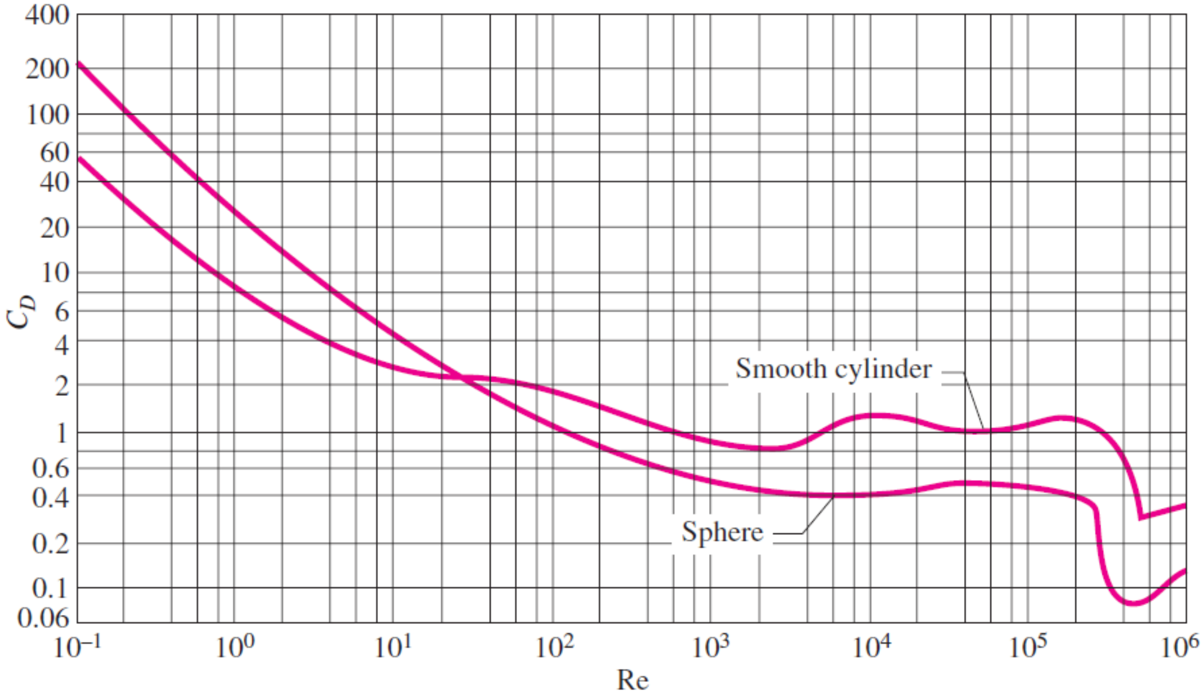


Figure 2.1. Average drag coefficient (C_D) for cross flow over a smooth circular cylinder and a smooth sphere

2.4.1 Effect of Surface Roughness

For a circular cylinder or sphere, an increase in the surface roughness may actually decrease the drag coefficient. Note that A is the frontal area (A=LD for a cylinder of length L, and $A=\pi D^2/4$ for a sphere). Where F_D is the drag force acting on the cylinder.

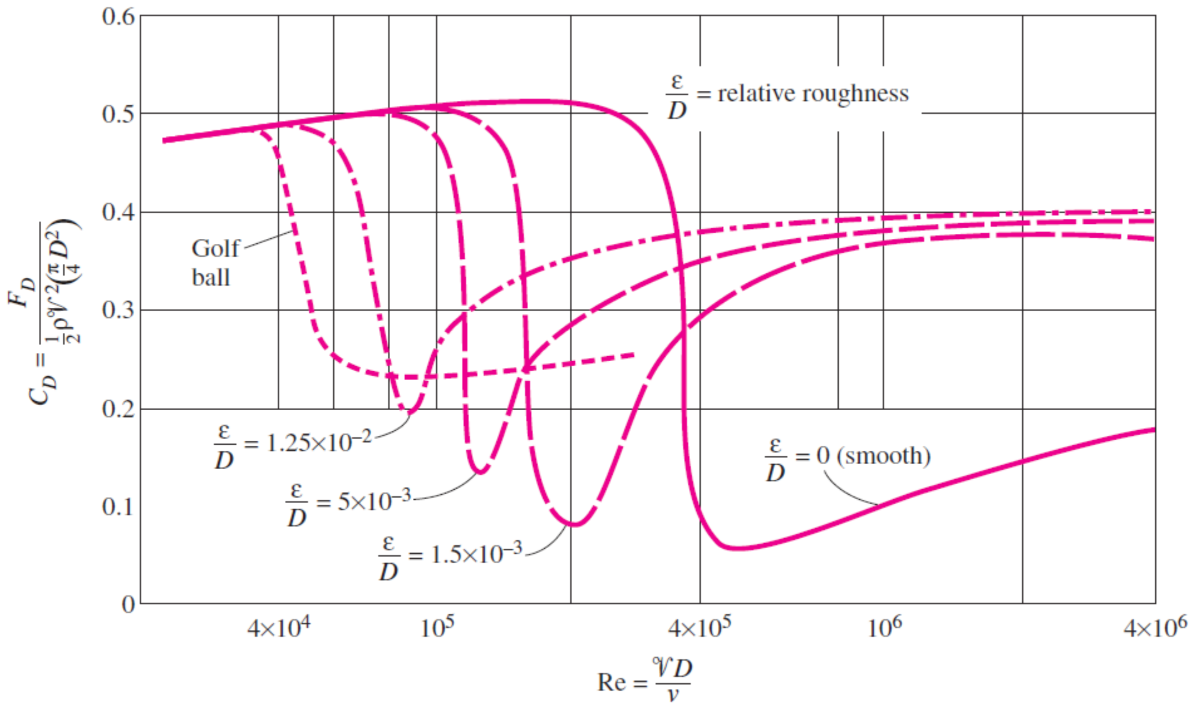


Figure 2.2. The effect of surface roughness on the drag coefficient of a sphere for turbulent flow.

2.4.2 Heat Transfer Coefficient

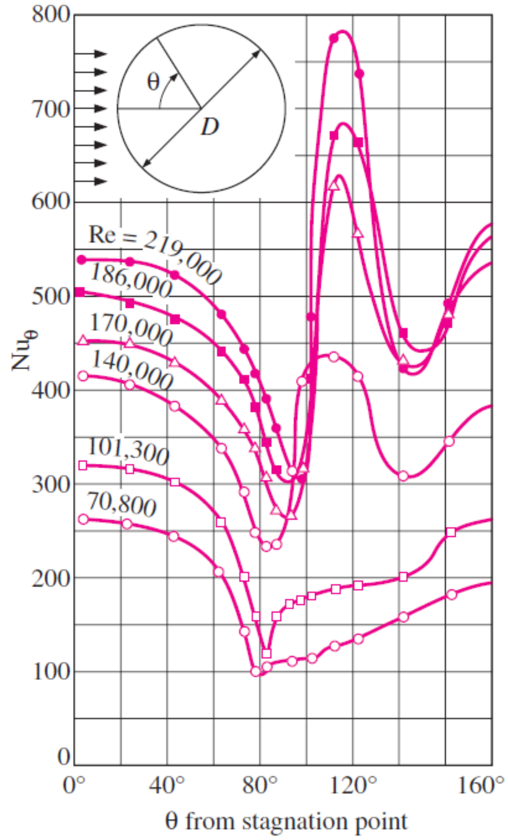


Figure 2.3. Variation of the local Nusselt number along the circumference of a circular cylinder in cross flow of air.

The average Nusselt number for cross flow over a *cylinder* is

$$Nu_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\, Re^{1/2}\, Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5} \tag{2.20}$$

This relation is quite comprehensive in that it correlates available data well for ($Re\, Pr > 0.2$). The fluid properties are evaluated at the film temperature.

For flow over a *sphere*, the following comprehensive correlation is recommended:

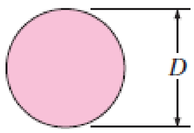
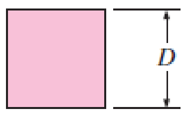
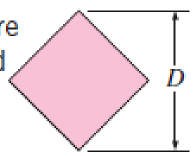
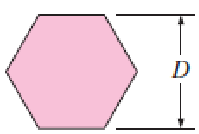
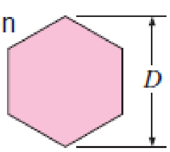
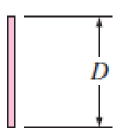
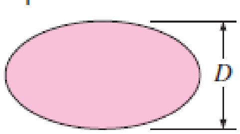
$$Nu_{sph} = \frac{hD}{k} = 2 + [0.4\, Re^{1/2} + 0.06\, Re^{2/3}]\, Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \qquad (2.21)$$

$$3.5 \leq Re \leq 80{,}000 \qquad 0.7 \leq Pr \leq 380$$

The fluid properties are evaluated at the free-stream temperature T_α, except for μ_s, which is evaluated at the surface temperature T_s.

TABLE 7-1

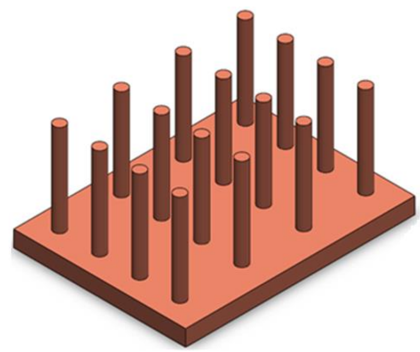
Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\,Pr^{1/3}$ $Nu = 0.911Re^{0.385}\,Pr^{1/3}$ $Nu = 0.683Re^{0.466}\,Pr^{1/3}$ $Nu = 0.193Re^{0.618}\,Pr^{1/3}$ $Nu = 0.027Re^{0.805}\,Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\,Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\,Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\,Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\,Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\,Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\,Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\,Pr^{1/3}$

2.5 Flow across Tube Banks

Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment such as the condensers and evaporators of power plants, refrigerators, and air conditioners. In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.

In a heat exchanger that involves a tube bank, the tubes are usually placed in a shell (and thus the name shell-and-tube heat exchanger), especially when the fluid is a liquid, and the fluid flows through the space between the tubes and the shell. Other types will be studied later.

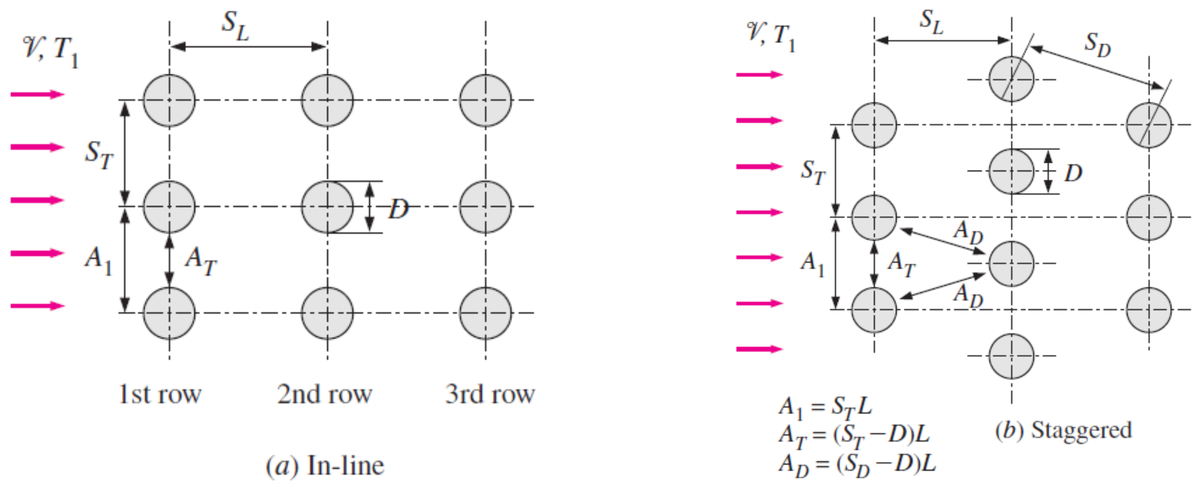


The tubes in a tube bank are usually arranged either in-line or staggered in the direction of flow. The outer tube diameter D is taken as the *characteristic length*. The arrangement of the tubes in the tube bank is characterized by the transverse pitch S_T, longitudinal pitch S_L, and the diagonal pitch S_D between tube centers. The *diagonal pitch* is determined from

$$S_D = \sqrt{S_L^2 + (S_T/2)^2} \qquad (2.23)$$

As the fluid enters the tube bank, the flow area decreases from $A_1=S_T \times L$ to $A_T=(S_T-D)L$ between the tubes, and thus flow velocity increases. In staggered arrangement, the velocity may increase further in the diagonal region if the tube rows are very close to each other. In tube banks, the flow characteristics are dominated by the maximum velocity V_{max} that occurs within the tube bank rather than the approach velocity *V*. Therefore, the Reynolds number is defined on the basis of maximum velocity as

$$\mathrm{Re}_D = \frac{\rho \mathcal{V}_{max} D}{\mu} = \frac{\mathcal{V}_{max} D}{\nu} \tag{2.24}$$



For in-line arrangement, the maximum velocity occurs at the minimum flow area between the tubes, $\rho V A_1 = \rho V_{max} A_T$ or $V S_T = V_{max}(S_T - D)$. Then the maximum velocity becomes

$$\mathcal{V}_{max} = \frac{S_T}{S_T - D} \mathcal{V} \tag{2.25}$$

In staggered arrangement,

$$\textit{Staggered and } S_D < (S_T + D)/2: \qquad \mathcal{V}_{max} = \frac{S_T}{2(S_D - D)} \mathcal{V} \tag{2.26}$$

Several correlations, all based on experimental data, have been proposed for the average Nusselt number for cross flow over tube banks. The general form is:

$$Nu_D = \frac{hD}{k} = C\, Re_D^m\, Pr^n (Pr/Pr_s)^{0.25} \qquad \begin{array}{l} 0.7 < Pr < 500 \\ 0 < Re_D < 2 \times 10^6 \end{array} \qquad (2.27)$$

The uncertainty in the values of Nusselt number obtained from these relations is ±15%. Note that all properties except Pr_s are to be evaluated at the arithmetic *mean temperature* of the fluid determined from

$$T_m = \frac{T_i + T_e}{2} \qquad (2.28)$$

where T_i and T_e are the fluid temperatures at the inlet and the exit of the tube bank, respectively. The *1ogarithmic mean temperature difference* ΔT_{ln} defined as

$$\Delta T_{ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)} \qquad (2.29)$$

TABLE 7–2

Nusselt number correlations for cross flow over tube banks for N > 16 and 0.7 < Pr < 500 (from Zukauskas, Ref. 15, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.27\, Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.033\, Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.35 (S_T/S_L)^{0.2}\, Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.031 (S_T/S_L)^{0.2}\, Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

The exit temperature of the fluid T_e can be determined from

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m} C_p}\right) \tag{2.30}$$

where $A_s = N\pi DL$, $\quad \dot{m} = \rho \mathcal{V}(N_T S_T L)$

Here N is the total number of tubes in the bank, N_T is the number of tubes in a transverse plane, L is the length of the tubes, and V is the velocity of the fluid just before entering the tube bank. Then the heat transfer rate can be determined from

$$\dot{Q} = hA_s \Delta T_{\ln} = \dot{m} C_p (T_e - T_i) \tag{2.31}$$

The pressure drop is estimated by

$$\Delta P = N_L f \chi \frac{\rho \mathcal{V}_{\max}^2}{2} \tag{2.32}$$

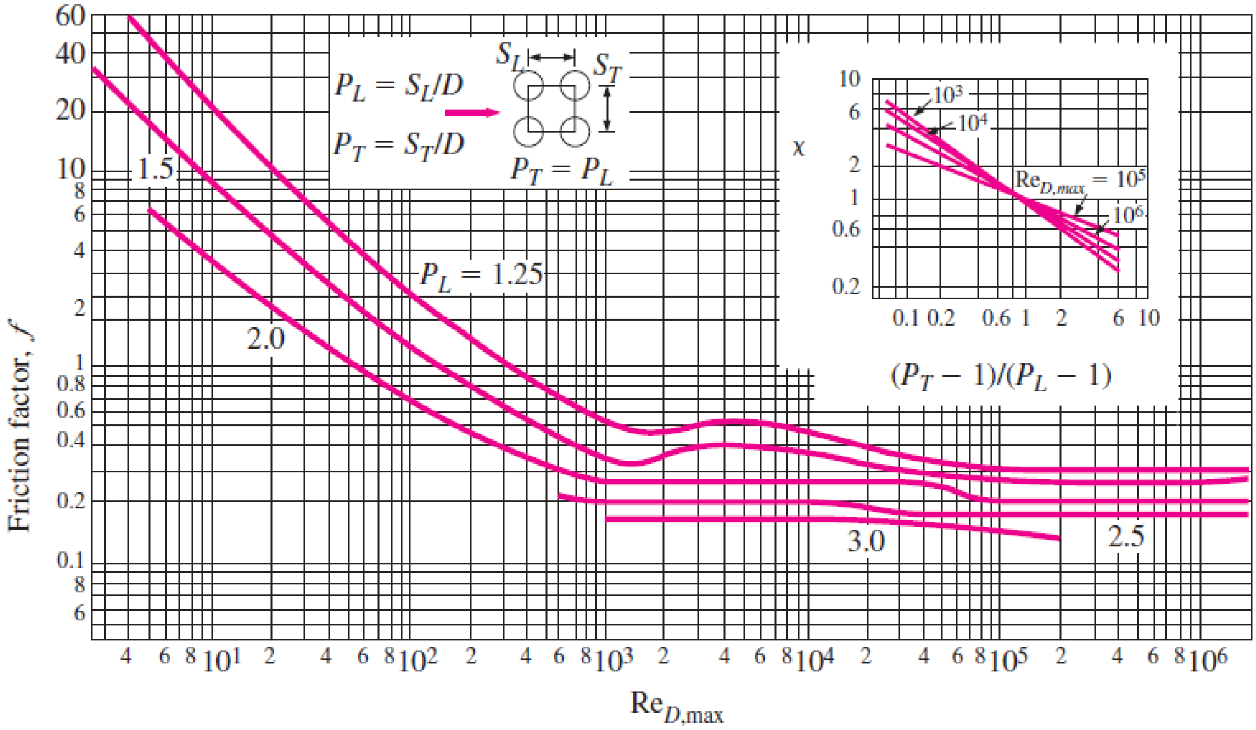
where f is the friction factor and X is the correction factor, both plotted in Figures 7–27a and 7–27b.

The pumping power required can be determined from

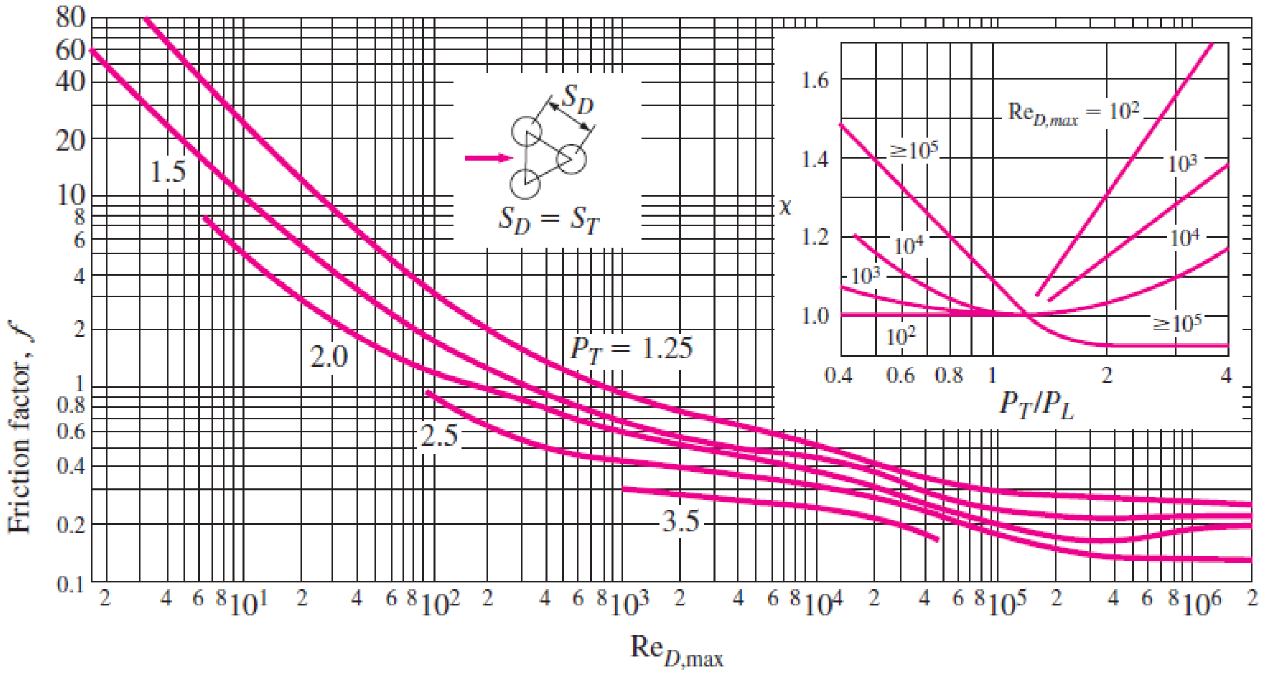
$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho} \tag{2.33}$$

$$\dot{V} = \mathcal{V}(N_T S_T L) \qquad \text{Volumetric flowrate} \tag{2.34}$$

$$\dot{m} = \rho \dot{V} = \rho \mathcal{V}(N_T S_T L) \qquad \text{Mass flow rate} \tag{2.35}$$



(a) In-line arrangement



(b) Staggered arrangement

Figure 2.4. The friction factor and X of the in-line and staggered arrangement according to the P_T and P_L.

The conclusion of the chapter:

The force a flowing fluid exerts on a body in the flow direction is called *drag*. The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag* since it is caused by frictional effects, and the part that is due directly to pressure is called the *pressure drag* or *form drag* because of its strong dependence on the form or shape of the body.

The *drag coefficient* C_D is a dimensionless number that represents the drag characteristics of a body, and is defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho \mathcal{V}^2 A}$$

where A is the *frontal area* for blunt bodies, and surface area for parallel flow over flat plates or thin airfoils. For flow over a flat plate, the Reynolds number is

$$\text{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{\nu}$$

Transition from laminar to turbulent occurs at the *critical Reynolds* number of

$$\text{Re}_{x,\,\text{cr}} = \frac{\rho \mathcal{V} x_{\text{cr}}}{\mu} = 5 \times 10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

Laminar: $C_{f,x} = \dfrac{0.664}{\text{Re}_x^{1/2}} \qquad \text{Re}_x < 5 \times 10^5$

$$\text{Nu}_x = \frac{h_x x}{k} = 0.332\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3} \qquad \text{Pr} > 0.6$$

Turbulent: $C_{f,x} = \dfrac{0.0592}{\text{Re}_x^{1/5}}, \qquad 5 \times 10^5 \le \text{Re}_x \le 10^7$

$$\text{Nu}_x = \frac{h_x x}{k} = 0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_x \le 10^7 \end{array}$$

The *average* friction coefficient relations for flow over a flat plate are:

Laminar: $C_f = \dfrac{1.328}{\text{Re}_L^{1/2}} \qquad \text{Re}_L < 5 \times 10^5$

Turbulent: $C_f = \dfrac{0.074}{\text{Re}_L^{1/5}} \qquad 5 \times 10^5 \le \text{Re}_L \le 10^7$

Combined: $C_f = \dfrac{0.074}{\text{Re}_L^{1/5}} - \dfrac{1742}{\text{Re}_L} \qquad 5 \times 10^5 \le \text{Re}_L \le 10^7$

Rough surface, turbulent: $C_f = \left(1.89 - 1.62 \log \dfrac{\varepsilon}{L}\right)^{-2.5}$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $\text{Nu} = \dfrac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} \qquad \text{Re}_L < 5 \times 10^5$

Turbulent:

$$\text{Nu} = \frac{hL}{k} = 0.037\,\text{Re}_L^{0.8}\,\text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_L \le 10^7 \end{array}$$

Combined:

$$\text{Nu} = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\text{Pr}^{1/3}, \qquad \begin{array}{l} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_L \le 10^7 \end{array}$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $\text{Nu}_x = \dfrac{\text{Nu}_{x\,(\text{for}\ \xi=0)}}{[1 - (\xi/x)^{3/4}]^{1/3}} = \dfrac{0.332\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3}}{[1 - (\xi/x)^{3/4}]^{1/3}}$

Turbulent: $\text{Nu}_x = \dfrac{\text{Nu}_{x\,(\text{for}\ \xi=0)}}{[1 - (\xi/x)^{9/10}]^{1/9}} = \dfrac{0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3}}{[1 - (\xi/x)^{9/10}]^{1/9}}$

Laminar: $h = \dfrac{2[1 - (\xi/x)^{3/4}]}{1 - \xi/L}\,h_{x=L}$

Turbulent: $h = \dfrac{5[1 - (\xi/x)^{9/10}]}{(1 - \xi/L)}\,h_{x=L}$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux*, the local Nusselt number is given by

Laminar: $\mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$

Turbulent: $\mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$

The average Nusselt numbers for cross flow over a *cylinder* and *sphere* are

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

which is valid for Re Pr > 0.2, and

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s, which is evaluated at the surface temperature T_s) in the case of a sphere.

In tube banks, the Reynolds number is based on the maximum velocity $\mathcal{V}_{\max}$ that is related to the approach velocity $\mathcal{V}$ as

In-line and *Staggered* with $S_D < (S_T + D)/2$:

$$\mathcal{V}_{\max} = \frac{S_T}{S_T - D}\mathcal{V}$$

Staggered with $S_D < (S_T + D)/2$:

$$\mathcal{V}_{\max} = \frac{S_T}{2(S_D - D)}\mathcal{V}$$

where S_T the transverse pitch and S_D is the diagonal pitch. The average Nusselt number for cross flow over tube banks is expressed as

$$\mathrm{Nu}_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\mathrm{Pr}^n(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$$

where the values of the constants C, m, and n depend on value Reynolds number. Such correlations are given in Table 7–2. All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid defined as $T_m = (T_i + T_e)/2$.

The average Nusselt number for tube banks with less than 16 rows is expressed as

$$\mathrm{Nu}_{D,N_L} = F\,\mathrm{Nu}_D$$

where F is the *correction factor* whose values are given in Table 7-3. The heat transfer rate to or from a tube bank is determined from

$$\dot{Q} = hA_s\Delta T_{\ln} = \dot{m}C_p(T_e - T_i)$$

where $\Delta T_{\ln}$ is the logarithmic mean temperature difference defined as

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

and the exit temperature of the fluid T_e is

$$T_e = T_s - (T_s - T_i)\exp\left(\frac{A_s h}{\dot{m}C_p}\right)$$

where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid. The pressure drop ΔP for a tube bank is expressed as

$$\Delta P = N_L f \chi \frac{\rho\mathcal{V}_{\max}^2}{2}$$

where f is the friction factor and χ is the correction factor, both given in Figs. 7–27.

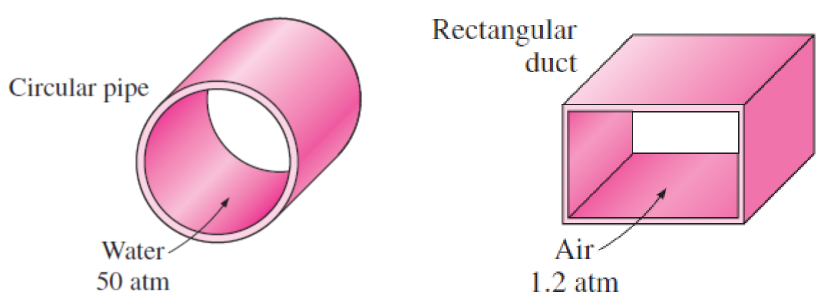
Chapter three

Internal Forced Convection

3.1 Introduction

Circular pipes are commonly used in wide engineering applications, and *noncircular* pipes are usually used in applications such as the heating and cooling systems of buildings where the *pressure difference* is relatively small and the manufacturing and installation costs are lower. The terms *pipe*, *duct*, *tube*, and *conduit* are usually used interchangeably for flow sections. In general, flow sections of circular cross section are referred to as *pipes* (especially when the fluid is a liquid), and the flow sections of noncircular cross section as *ducts* (especially when the fluid is a gas).

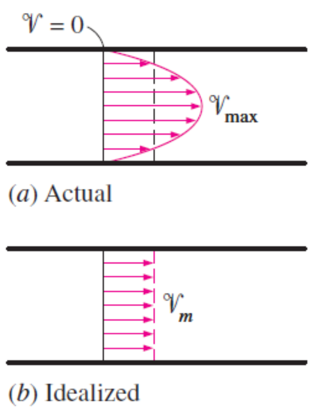
Small diameter pipes are usually referred to as *tubes*. Thus, we will use more descriptive phrases (such as a circular pipe or a rectangular duct) to avoid any misunderstandings.



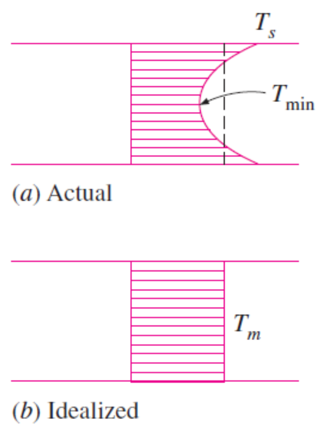
3.2 Mean Velocity and Mean Temperature

The fluid velocity in a tube changes from zero at the surface because of the no-slip condition, to a maximum at the tube center. Therefore, it is convenient to work with an *average* or *mean velocity* V_m, which remains constant for *incompressible* flow when the cross sectional area of the tube is constant.

When a fluid is heated or cooled as it flows through a tube, the temperature of the fluid at any cross section changes from T_s at the surface of the wall to some maximum (or minimum in the case of heating) at the tube center. In fluid flow, it is convenient to work with an *average* or *mean temperature* T_m that remains uniform at a cross section. Unlike the mean velocity, the mean temperature T_m will change in the flow direction whenever the fluid is heated or cooled.



Velocity profile



Temperature profile

3.3 Laminar Flow in Tubes

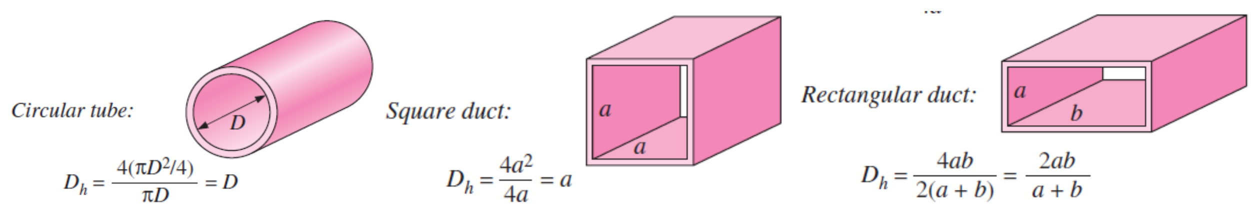
Transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some range of velocity where the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent. Most pipe flows encountered in practice are *turbulent*. Laminar flow is encountered when *highly viscous fluids* such as oils flow in small diameter tubes or narrow passages. For flow in a circular tube, the Reynolds number is defined as

$$Re = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\nu} \tag{3.1}$$

For flow through noncircular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the *hydraulic diameter* D_h defined as

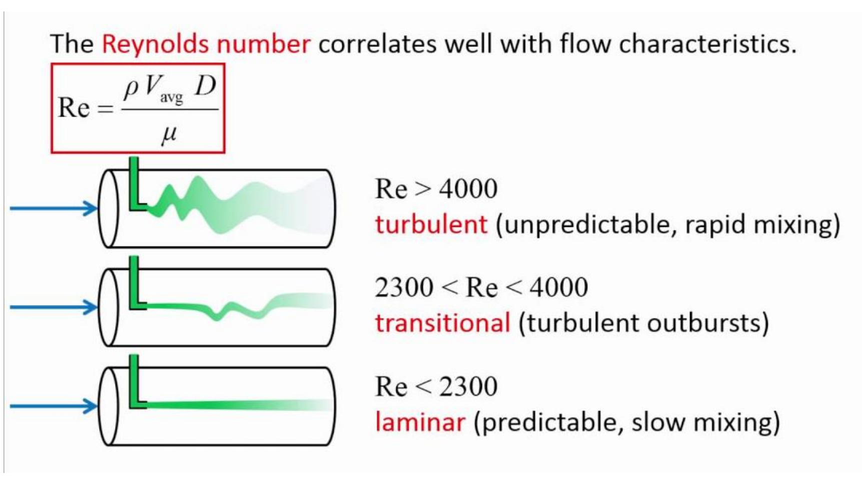
$$D_h = \frac{4A_c}{p} \tag{3.2}$$

where A_c is the cross sectional area of the tube and p is its perimeter.



It is desirable to have precise values of Reynolds numbers for laminar, transitional, and turbulent flows, which are:

$Re < 2300$	laminar flow
$2300 \le Re \le 10{,}000$	transitional flow
$Re > 10{,}000$	turbulent flow



For laminar flows: the *velocity profile* is obtained to be:

$$\mathcal{V}(r) = 2\mathcal{V}_m\left(1 - \frac{r^2}{R^2}\right) \quad \text{For } r=0, \quad \mathcal{V}_{max} = 2\mathcal{V}_m \tag{3.3}$$

The pressure drop along the tube is:

$$\Delta P = \frac{8\mu L\,\mathcal{V}_m}{R^2} = \frac{32\mu L\,\mathcal{V}_m}{D^2} \quad \text{or} \quad \Delta P = f\frac{L}{D}\frac{\mu\mathcal{V}_m^2}{2} \tag{3.4}$$

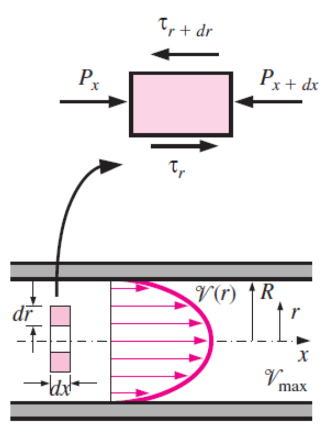
where the dimensionless quantity f is the *friction factor* (Darcy friction factor). It should not be confused with the *friction coefficient* C_f (Fanning friction factor), which is defined as

$$C_f = \tau_s(\rho\mathcal{V}_m^2/2) = f/4 \tag{3.5}$$

The friction factor for the *fully developed laminar* flow in a *circular tube* is

$$f = \frac{64\mu}{\rho D\mathcal{V}_m} = \frac{64}{\text{Re}} \tag{3.6}$$

This equation shows that in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the tube surface.



The tempeature profile for the laminar flow is:

T_s=constant

$$\text{Nu} = \frac{hD}{k} = 3.66$$

$$\text{Nu} = 3.66 + \frac{0.065\,(D/L)\,\text{Re}\,\text{Pr}}{1 + 0.04[(D/L)\,\text{Re}\,\text{Pr}]^{2/3}}$$

For thermal entrance region

q_s=constant

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right)$$

$$T_m = T_s - \frac{11}{24}\frac{\dot{q}_s R}{k}$$

$$h = \frac{24}{11}\frac{k}{R} = \frac{48}{11}\frac{k}{D} = 4.36\frac{k}{D}$$

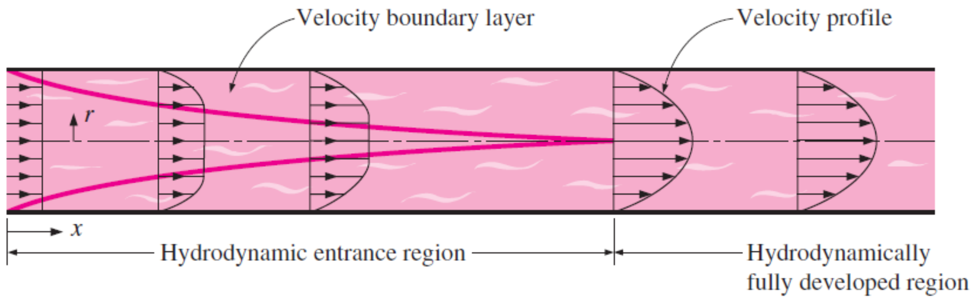
$$\text{Nu} = \frac{hD}{k} = 4.36$$

(3.7)

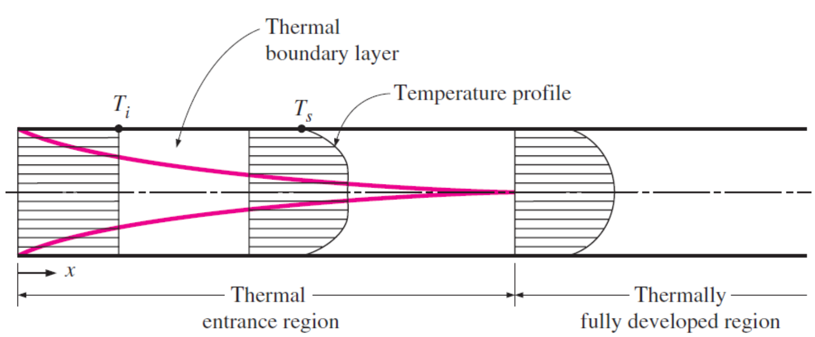
3.4 The Entrance Region

The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the *hydrodynamic entrance region*, and the length of this region is called the *hydrodynamic entry length L_h*.

Flow in the entrance region is called *hydrodynamically developing flow* since this is the region where the velocity profile develops. The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the *hydrodynamically fully developed region*. The velocity profile in the fully developed region is *parabolic* in laminar flow and *somewhat flatter* in turbulent flow due to eddy motion in radial direction.



The region of flow over which the thermal boundary layer develops and reaches the tube center is called the *thermal entrance region*, and the length of this region is called the *thermal entry length* L_t. Flow in the thermal entrance region is called *thermally developing flow* since this is the region where the temperature profile develops. The region beyond the thermal entrance region is called the *thermally fully developed region*. The region in which the flow is both hydrodynamically and thermally developed and thus both the velocity and dimensionless temperature profiles remain unchanged is called *fully developed flow*.



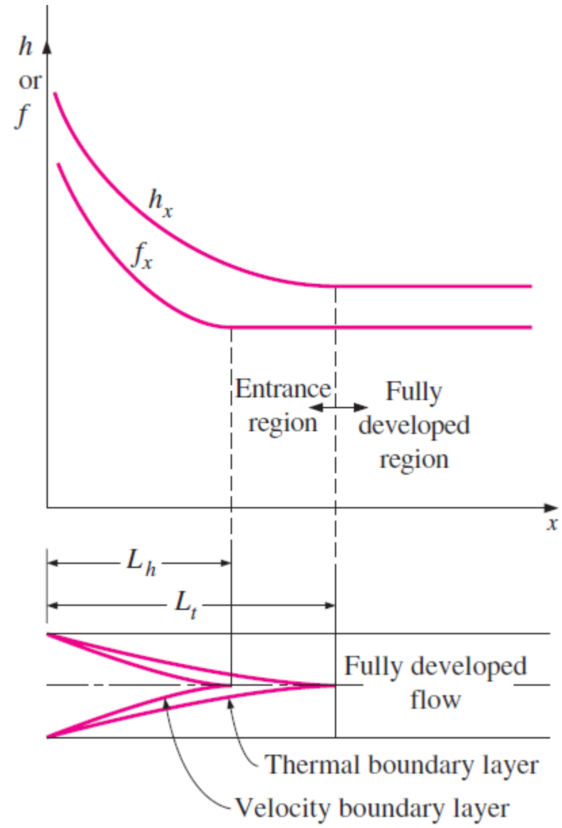
3.5 Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the tube entrance. In laminar flow, the hydrodynamic and thermal entry lengths are given approximately as

$$L_{h,\ \text{laminar}} \approx 0.05\ \text{Re}\ D$$
$$L_{t,\ \text{laminar}} \approx 0.05\ \text{Re Pr}\ D = \text{Pr}\ L_{h,\ \text{laminar}} \qquad (3.8)$$

In turbulent flow, the hydrodynamic and thermal entry lengths are of about the same size and independent of the Prandtl number. Also, the friction factor and the heat transfer coefficient remain constant in fully developed laminar or turbulent flow since the velocity and temperature profiles do not vary in the flow direction.

$$L_{h,\ \text{turbulent}} = 1.359\ \text{Re}^{1/4} \qquad (3.9)$$



The hydrodynamic entry length is much shorter in turbulent flow, as expected, and its dependence on the Reynolds number is weaker.

$$L_{h,\ \text{turbulent}} \approx L_{t,\ \text{turbulent}} \approx 10D \qquad (3.10)$$

It must be noted that

- The Nusselt numbers are much higher in the entrance region.
- The Nusselt number reaches a constant value and the flow can be assumed to be fully developed for x>10D.
- The Nusselt numbers for q_s=c and T_s=c are identical in the fully developed regions, and nearly identical in the entrance regions.

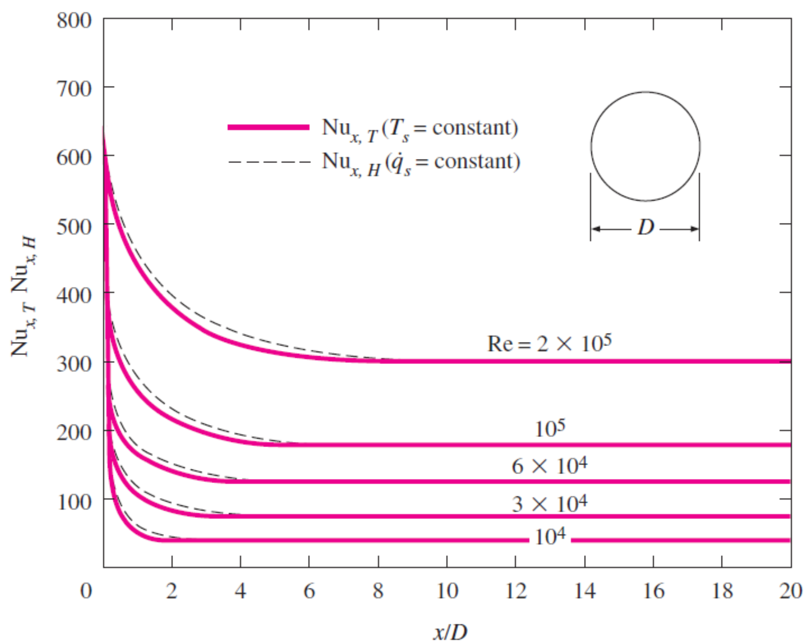
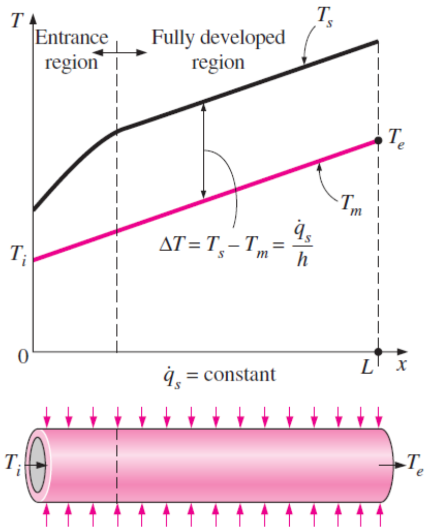


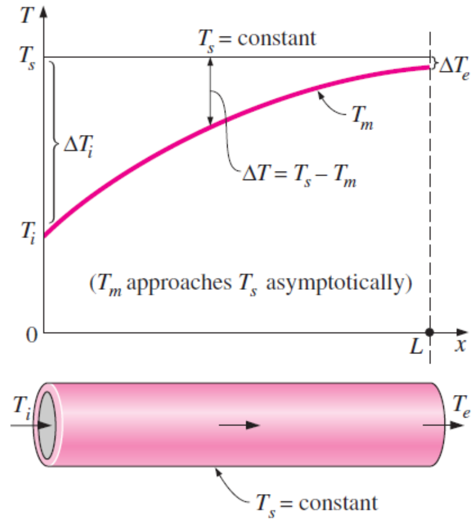
Figure 3.1. Local Nusselt number along the circular tube for both thermal boundary conditions.

3.6 Thermal boundary condition at the heated wall surface

In fully developed flow in a tube subjected to *constant surface heat flux*, the temperature gradient is independent of x and thus the shape of the temperature profile does not change along the tube.



$$\dot{Q} = \dot{q}_s A_s = \dot{m}C_p(T_e - T_i) \qquad \text{(W)}$$



$$\dot{Q} = hA_s\Delta T_{ave} = hA_s(T_s - T_m)_{ave} \qquad \text{(W)}$$

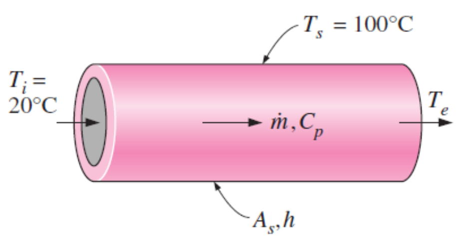
$$T_m = \frac{T_e + T_i}{2}$$

$$\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}C_p} \qquad \text{or}$$

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$

For the constant surface temperature condition;

Note that the *temperature difference* between the fluid and the surface decays exponentially in the flow direction, and the rate of decay depends on the magnitude of the exponent $hA_x/\dot{m}C_p$, as shown above. This *dimensionless parameter* is called the *number of transfer units*, denoted by NTU, and is: *a measure of the effectiveness of the heat transfer systems*. For NUT > 5, the exit temperature of the fluid becomes almost equal to the surface temperature, $T_e \approx T_s$. Noting that the fluid temperature can approach the surface temperature but not greater, an NTU = 5 indicates that the limit is reached for heat transfer, and the heat transfer will not increase no matter how much we extend the length of the tube. A small value of NTU, indicates more opportunities for heat transfer, and the heat transfer will continue increasing as the tube length is increased. A large NTU and thus a large heat transfer surface area (which means a large tube) may be desirable from a heat transfer point of view, *but it may be unacceptable from an economic point of view*. The selection of heat transfer equipment usually reflects a compromise between heat transfer performance and cost.



NTU = $hA_s / \dot{m}C_p$	T_e, °C
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

FIGURE 8–15
An NTU greater than 5 indicates that the fluid flowing in a tube will reach the surface temperature at the exit regardless of the inlet temperature.

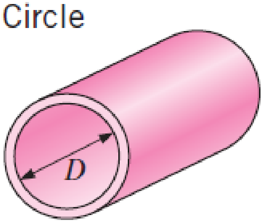
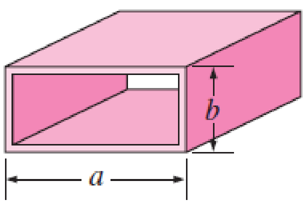
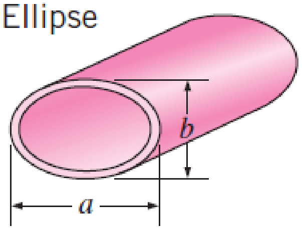
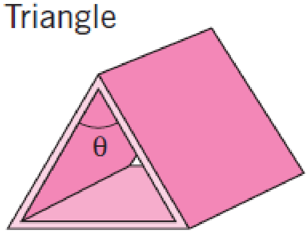
The logarithmic mean temperature difference relation is

$$\text{LMTD} = \frac{\Delta T_{max} - \Delta T_{min}}{\ln \dfrac{\Delta T_{max}}{\Delta T_{min}}}$$

$$\Delta T_{\ln} = \frac{(T_s - T_i) - (T_s - T_e)}{\ln\left[\dfrac{(T_s - T_i)}{(T_s - T_e)}\right]} \tag{3.11}$$

TABLE 8–1

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = \mathcal{V}_m D_h/\nu$, and $Nu = hD_h/k$)

Tube Geometry	a/b or $\theta°$	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

The average Nusselt number for the thermal entrance region of flow between isothermal parallel plates of length L is expressed as

$$\mathrm{Nu} = 7.54 + \frac{0.03\,(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.016[(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}} \qquad (3.12)$$

where D_h is the hydraulic diameter, which is twice the spacing of the plates (D_h =2H). This relation can be used for Re ≤ 2800.

3.7 Turbulent Flow in Tubes

Turbulent flow is commonly utilized in practice because of the higher heat transfer coefficients associated with it.

For *smooth tubes*, the friction factor in turbulent flow can be determined from the explicit first *Petukhov* equation

Smooth tubes: $$f = (0.790 \ln \mathrm{Re} - 1.64)^{-2} \qquad 10^4 < \mathrm{Re} < 10^6 \qquad (3.13)$$

The Nusselt number equations are:

$$\mathrm{Nu} = 0.125\, f\, \mathrm{Re}\mathrm{Pr}^{1/3} \qquad f = 0.184\, \mathrm{Re}^{-0.2}$$ Chilton–Colburn analogy

Thus we obtain,

$$\mathrm{Nu} = 0.023\, \mathrm{Re}^{0.8}\, \mathrm{Pr}^{1/3} \qquad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re} > 10{,}000 \end{pmatrix}$$ Colburn Eq.

Its accuracy was improved to be

$$\mathrm{Nu} = 0.023\, \mathrm{Re}^{0.8}\, \mathrm{Pr}^{n}$$ n=0.4 for heating, n=0.3 for cooling — Dittus-Boelter Eq. 25% error

$$\mathrm{Nu} = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 10^4 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix}$$ 2nd Petukhov Eq. 10% error

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix}$$ Gnielinski Eq.

(3.14)

For liquid metals;

Liquid metals, T_s = constant: $\quad Nu = 4.8 + 0.0156\,Re^{0.85}\,Pr_s^{0.93}$

Liquid metals, $\dot{q}_s$ = constant: $\quad Nu = 6.3 + 0.0167\,Re^{0.85}\,Pr_s^{0.93}$

(0.004<Pr<0.01)

(3.15)

3.7.1 Rough Surfaces

The friction factor in *fully developed turbulent flow* depends on the Reynolds number and the relative roughness ε/D.

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$ Colebrook Eq. (3-16)

$$\frac{1}{\sqrt{f}} \approx -1.8\log\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$ Haaland Eq. (3-17)

In 1944, L. F. Moody plotted this formula into the famous *Moody chart* given in the Appendix. It presents the friction factors for pipe flow as a function of the Reynolds number and ε/D over a wide range. Although the Moody chart is developed for *circular pipes*, it can also be used for *noncircular pipes* by replacing the diameter by the hydraulic diameter.

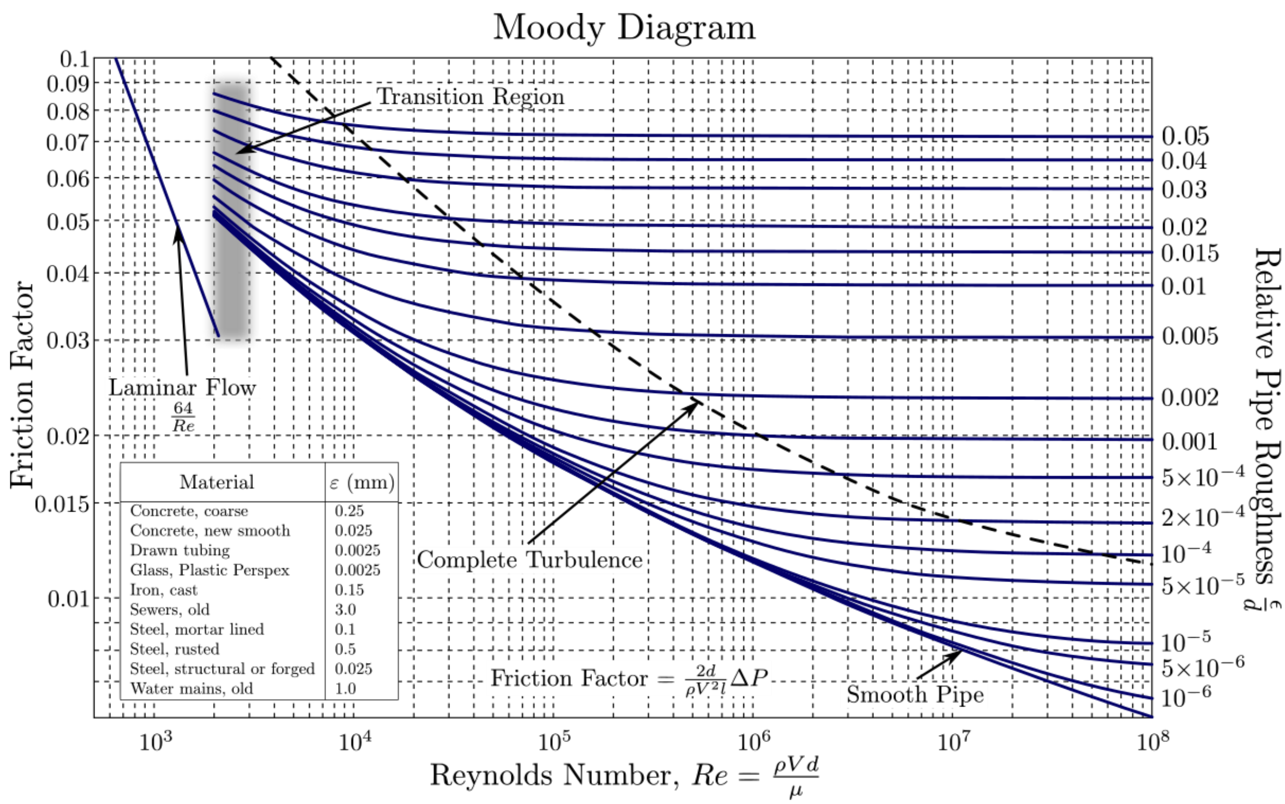


TABLE 8–3

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

The chapter conclusion:

Internal flow is characterized by the fluid being completely confined by the inner surfaces of the tube. The mean velocity and mean temperature for a circular tube of radius R are expressed as

$$\mathcal{V}_m = \frac{2}{R^2}\int_0^R \mathcal{V}(r,x)rdr \qquad \text{and} \qquad T_m = \frac{2}{\mathcal{V}_m R^2}\int_0^R \mathcal{V}Trdr$$

The Reynolds number for internal flow and the hydraulic diameter are defined as

$$\text{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\nu} \qquad \text{and} \qquad D_h = \frac{4A_c}{p}$$

The flow in a tube is laminar for Re < 2300, turbulent for Re > 10,000, and transitional in between.

The length of the region from the tube inlet to the point at which the boundary layer merges at the centerline is the *hydrodynamic entry length* L_h. The region beyond the entrance region in which the velocity profile is fully developed is the *hydrodynamically fully developed region*. The length of the region of flow over which the thermal boundary layer develops and reaches the tube center is the *thermal entry length* L_t. The region in which the flow is both hydrodynamically and thermally developed is the *fully developed flow region*. The entry lengths are given by

$$L_{h,\,\text{laminar}} \approx 0.05\,\text{Re}\,D$$
$$L_{t,\,\text{laminar}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = \text{Pr}\,L_{h,\,\text{laminar}}$$
$$L_{h,\,\text{turbulent}} \approx L_{t,\,\text{turbulent}} \approx 10D$$

For $\dot{q}_s$ = constant, the rate of heat transfer is expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m}C_p(T_e - T_i)$$

For T_s = constant, we have

$$\dot{Q} = hA_s\Delta T_{\ln} = \dot{m}C_p(T_e - T_i)$$
$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$
$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

The pressure drop and required pumping power for a volume flow rate of $\dot{V}$ are

$$\Delta P = \frac{L}{D}\frac{\rho \mathcal{V}_m^2}{2} \qquad \text{and} \qquad \dot{W}_{\text{pump}} = \dot{V}\Delta P$$

For *fully developed laminar flow* in a circular pipe, we have:

$$\mathcal{V}(r) = 2\mathcal{V}_m\left(1 - \frac{r^2}{R^2}\right) = \mathcal{V}_{\max}\left(1 - \frac{r^2}{R^2}\right)$$
$$f = \frac{64\mu}{\rho D\mathcal{V}_m} = \frac{64}{\text{Re}}$$
$$\dot{V} = \mathcal{V}_{\text{ave}}A_c = \frac{\Delta PR^2}{8\mu L}\pi R^2 = \frac{\pi R^4\,\Delta P}{8\mu L} = \frac{\pi R^4\,\Delta P}{128\mu L}$$

Circular tube, laminar ($\dot{q}_s$ = constant): $\quad \text{Nu} = \frac{hD}{k} = 4.36$

Circular tube, laminar (T_s = constant): $\quad \text{Nu} = \frac{hD}{k} = 3.66$

For *developing laminar flow* in the entrance region with constant surface temperature, we have

Circular tube: $\quad \text{Nu} = 3.66 + \dfrac{0.065(D/L)\,\text{Re}\,\text{Pr}}{1 + 0.04[(D/L)\,\text{Re}\,\text{Pr}]^{2/3}}$

Circular tube: $\quad \text{Nu} = 1.86\left(\dfrac{\text{Re}\,\text{Pr}\,D}{L}\right)^{1/3}\left(\dfrac{\mu_b}{\mu_s}\right)^{0.14}$

Parallel plates: $\quad \text{Nu} = 7.54 + \dfrac{0.03(D_h/L)\,\text{Re}\,\text{Pr}}{1 + 0.016[(D_h/L)\,\text{Re}\,\text{Pr}]^{2/3}}$

For *fully developed turbulent flow with smooth surfaces,* we have

$$f = (0.790\ln \text{Re} - 1.64)^{-2} \qquad 10^4 < \text{Re} < 10^6$$
$$\text{Nu} = 0.125f\,\text{Re}\,\text{Pr}^{1/3}$$
$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{1/3} \qquad \begin{pmatrix}0.7 \le \text{Pr} \le 160\\ \text{Re} > 10{,}000\end{pmatrix}$$

$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^n$ with $n = 0.4$ for *heating* and 0.3 for *cooling* of fluid

$$\text{Nu} = \frac{(f/8)(\text{Re} - 1000)\,\text{Pr}}{1 + 12.7(f/8)^{0.5}(\text{Pr}^{2/3} - 1)} \qquad \begin{pmatrix}0.5 \le \text{Pr} \le 2000\\ 3\times 10^3 < \text{Re} < 5\times 10^6\end{pmatrix}$$

The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$. For liquid metal flow in the range of $10^4 < \text{Re} < 10^6$ we have:

T_s = constant: $\quad \text{Nu} = 4.8 + 0.0156\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$

$\dot{q}_s$ = constant: $\quad \text{Nu} = 6.3 + 0.0167\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$

For *fully developed turbulent flow with rough surfaces,* the friction factor f is determined from the Moody chart or

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \approx -1.8\log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

For a *concentric annulus,* the hydraulic diameter is $D_h = D_o - D_i$, and the Nusselt numbers are expressed as

$$\text{Nu}_i = \frac{h_i D_h}{k} \qquad \text{and} \qquad \text{Nu}_o = \frac{h_o D_h}{k}$$

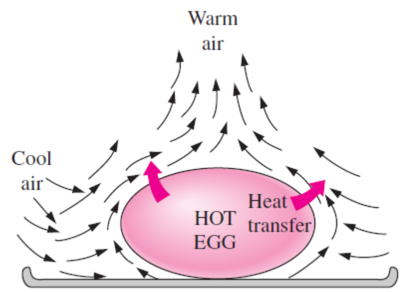
where the values for the Nusselt numbers are given in Table 8–4.

Chapter four

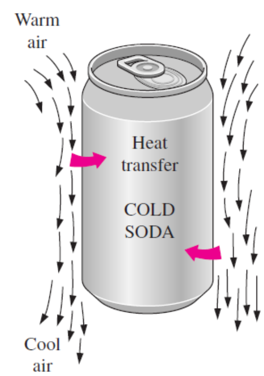
Natural Convection

4.1 Introduction

Many familiar heat transfer applications involve natural convection such as cooling of electronic equipment such (power transistors, TVs, and VCRs); electric baseboard heaters or steam radiators; refrigeration coils and power transmission lines; and bodies of animals and human beings.



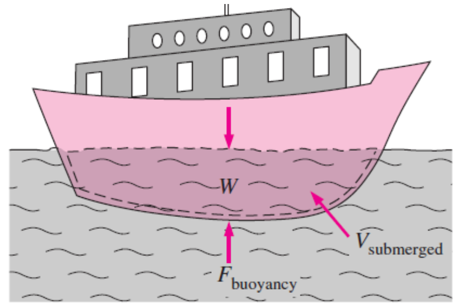
The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a *natural convection current*, and the heat transfer that is enhanced as a result of this natural convection current is called *natural convection heat transfer*.



In a gravitational field, there is a net force that pushes upward a light fluid placed in a heavier fluid. The upward force exerted by a fluid on a body completely or partially immersed in it is called the *buoyancy force*. The magnitude of the buoyancy force is equal to the weight of the fluid displaced by the body. That is,

$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, g V_{\text{body}}$$

where ρ_{fluid} is the average density of the fluid, g is the gravitational acceleration, and V_{body} is the volume of the portion of the body immersed in the fluid (the total volume of the body). In the absence of other forces, the net vertical force acting on a body is the difference between the weight of the body and the buoyancy force. That is,



$$F_{net} = W - F_{buoyancy} = \rho_{body}\, g V_{body} - \rho_{fluid}\, g V_{body} = (\rho_{body} - \rho_{fluid})\, g V_{body}$$

Note that this force is proportional to the difference in the densities of the fluid and the body immersed in it.

In heat transfer studies, the primary variable is temperature, and it is desirable to express the net buoyancy force in terms of temperature differences. But this requires expressing the density difference in terms of a temperature difference, which requires a knowledge of a property that represents the *variation of the density of a fluid with temperature at constant pressure*. The property that provides that information is the volume expansion coefficient β, defined as

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \quad (1/\mathrm{K}) \quad \text{or} \quad \beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \quad (\text{at constant } P)$$

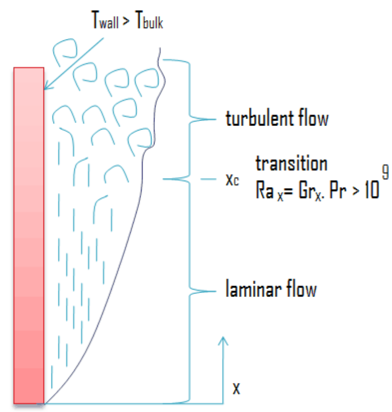
$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \qquad (\text{at constant } P)$$

where ρ_α is the density and T_α is the temperature of the fluid away from the surface.

Easily, the volume expansion coefficient β of an *ideal gas* (*P* = *ρ R T*) at a temperature T is equivalent to the inverse of the temperature:

$$\beta_{ideal\ gas} = \frac{1}{T} \qquad (1/\mathrm{K}) \qquad (4.1)$$

The smooth and parallel lines indicate that the flow is laminar, whereas the eddies and irregularities indicate that the flow is turbulent. Note that the lines are closest near the surface, indicating a higher temperature gradient.



4.2 Equation of Motion and the Grashof Number

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g$$

conservation of momentum in the x-direction

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g$$

due to u=0

thus

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

The dimensionless parameter, which is called the Grashof number Gr_L, represents the natural convection effects

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \qquad (4.2)$$

Where,

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s

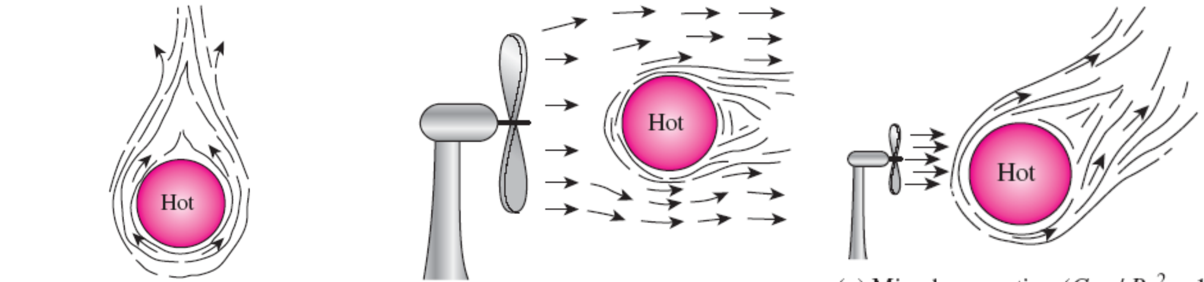
The flow regime in natural convection is governed by the dimensionless *Grashof number*, which represents: *the ratio of the buoyancy force to the viscous force acting on the fluid*.

The role played by the *Reynolds number* in forced convection is played by the *Grashof number* in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection. For vertical plates, for example, the critical Grashof number is observed to be about 10^9.

When a surface is subjected to external flow, the problem involves both natural and forced convection (Mixed Convection). The relative importance of each mode of heat transfer is determined by the value of the coefficient

$$Ri = \frac{Gr_L}{Re_L^2} \qquad \text{(Richardson number)} \qquad (4.3)$$

When Natural convection effects are negligible if $Ri \ll 1$, free convection dominates and the forced convection effects are negligible if $Ri \gg 1$, and both effects are significant and must be considered if $Ri = 1$.



(a) Natural convection ($Gr_L / Re_L^2 >> 1$) (b) Forced convection ($Gr_L / Re_L^2 << 1$) (c) Mixed convection ($Gr_L / Re_L^2 \approx 1$)

4.3 Natural Convection over Surfaces

Natural convection heat transfer on a surface depends on the geometry of the surface, its orientation, the variation of temperature on the surface, and the thermophysical properties of the fluid. Thus, the simple empirical correlations for the average Nusselt number (*Nu*) in natural convection is

$$\text{Nu} = \frac{hL_c}{k} = C(\text{Gr}_L\,\text{Pr})^n = C\,\text{Ra}_L^n \qquad (4.4)$$

where Ra_L is the Rayleigh number, which is :

$$\text{Ra}_L = \text{Gr}_L\,\text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\text{Pr} \qquad (4.5)$$

The values of the constants *C* and *n* depend on the geometry of the surface and the flow regime, which is characterized by the range of the Rayleigh number. Where (n=1/4) for laminar flow and (n=1/3) for turbulent flow. The value of the constant C is normally less than 1.

For a vertical flat plate (T_s=constant), the characteristic length is the plate height L. In Table 9–1 several relations for the average Nusselt number for an isothermal vertical plate are given.

In the case of (q_s=constant), the rate of heat transfer is known ($Q=q_s A_s$), but the surface temperature T_s is not. In fact, T_s increases with height along the plate. It turns out that the Nusselt number relations for the constant surface temperature and constant surface heat flux cases are *nearly identical*. Therefore, the relations for isothermal plates can also be used for plates subjected to uniform heat flux, provided that the plate midpoint temperature $T_L/2$ is used for T_s in the evaluation of the film temperature, Rayleigh number, and the Nusselt number.

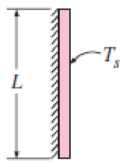
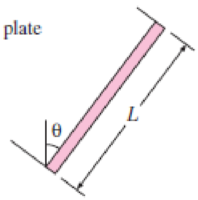
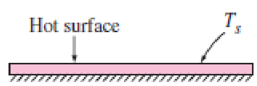
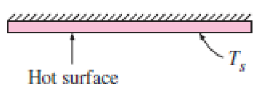
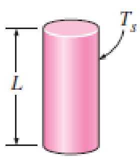
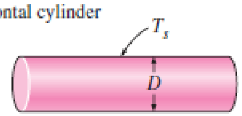
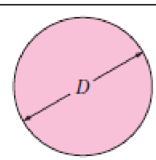
$$\text{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \tag{4.6}$$

For Vertical Cylinders, an outer surface of a vertical cylinder can be treated as a vertical plate when the diameter of the cylinder is sufficiently large so that the curvature effects are negligible. This condition is satisfied if

$$D \geq \frac{35L}{\text{Gr}_L^{1/4}} \tag{4.7}$$

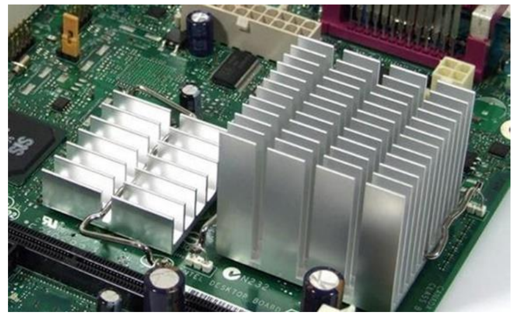
TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

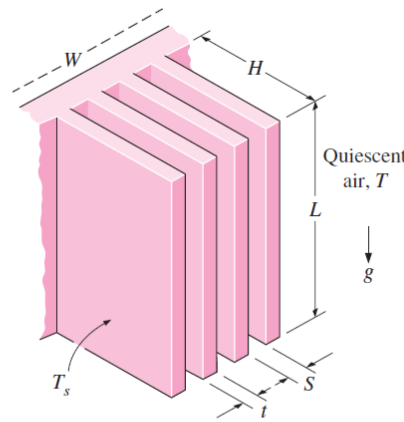
Geometry	Characteristic length L_c	Range of Ra	Nu	
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ $\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)	(9-19) (9-20) (9-21)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $\text{Ra} < 10^9$	
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ $\text{Nu} = 0.27\text{Ra}_L^{1/4}$	(9-22) (9-23) (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{\text{Gr}_L^{1/4}}$	
Horizontal cylinder 	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \dfrac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$	(9-25)
Sphere 	D	$\text{Ra}_D \leq 10^{11}$ $(\text{Pr} \geq 0.7)$	$\text{Nu} = 2 + \dfrac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$	(9-26)

4.4 Natural Convection from Finned Surfaces and Printed Circuit Board (PCB)

4.4.1 Cooling T_s=constant



Finned surfaces of various shapes, called heat sinks, are frequently used in the cooling of electronic devices. Energy dissipated by these devices is transferred to the heat sinks by conduction and from the heat sinks to the ambient air by natural or forced convection, depending on the power dissipation requirements. A properly selected heat sink may considerably lower the operation temperature of the components and thus reduce the risk of failure.



The characteristic length for vertical parallel plates used as fins is usually taken to be the spacing between adjacent fins S, although the fin height L could also be used. The Rayleigh number is expressed as

$$Ra_S = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2} Pr \qquad \text{and} \qquad Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr \tag{4.8}$$

The recommended relation for the average Nusselt number for vertical isothermal parallel plates is

$$T_s = \text{constant:} \qquad Nu = \frac{hS}{k} = \left[\frac{576}{(Ra_S S/L)^2} + \frac{2.873}{(Ra_S S/L)^{0.5}}\right]^{-0.5} \tag{4.9}$$

When the fins are essentially isothermal and the fin thickness *t* is small relative to the fin spacing S, the optimum fin spacing for a vertical heat sink is determined by Bar-Cohen and Rohsenow to be

$$S_{opt} = 2.714\left(\frac{S^3 L}{Ra_S}\right)^{0.25} = 2.714\,\frac{L}{Ra_L^{0.25}} \qquad T_s = \text{constant} \qquad (4.10)$$

$$Nu = \frac{h S_{opt}}{k} = 1.307 \qquad S = S_{opt} \qquad (4.11)$$

$$\dot{Q} = h(2nLH)(T_s - T_\infty) \qquad \text{Heat transfer rate} \qquad (4.12)$$

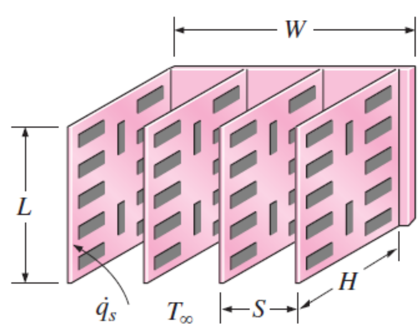
where $n = W/(S + t) \approx W/S$ is the number of fins on the heat sink

All fluid properties are estimated at the average temperature:

$$T_{ave} = (T_s + T_\alpha)/2 \qquad (4.13)$$

4.4.2 Natural Convection Cooling of Vertical PCB (q_s=constant)

The plate temperature in this case increases with height, reaching a maximum at the upper edge of the board. The modified *Rayleigh number* for uniform heat flux on both plates is expressed as



$$Ra_S^* = \frac{g\beta \dot{q}_s S^4}{k\nu^2} Pr \qquad (4.14)$$

The Nusselt number at the upper edge of the plate where maximum temperature occurs is determined from

$$\mathrm{Nu}_L = \frac{h_L S}{k} = \left[\frac{48}{\mathrm{Ra}_S^* S/L} + \frac{2.51}{(\mathrm{Ra}_L^* S/L)^{0.4}}\right]^{-0.5} \quad (4.15)$$

The optimum fin spacing for the case of uniform heat flux on both plates is given as

$$S_{\mathrm{opt}} = 2.12\left(\frac{S^4 L}{\mathrm{Ra}_S^*}\right)^{0.2} \qquad \dot{q}_s = \text{constant} \quad (4.16)$$

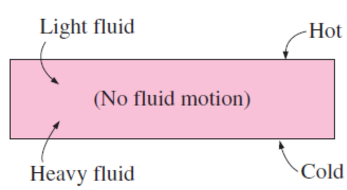
$$\dot{Q} = \dot{q}_s A_s = \dot{q}_s(2nLH) \qquad \text{Heat trasnfer rate} \quad (4.17)$$

$$\dot{q}_s = h_L(T_L - T_\infty) \quad (4.18)$$

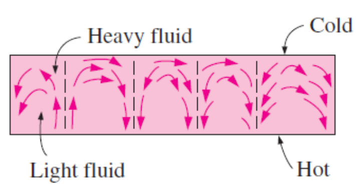
All fluid properties are estimated at the average temperature, eq. 13.

4.5 Natural Convection inside Enclosures

Its common application is through the windows. We certainly would insulate the windows to conserve energy. The problem is finding a transparent insulating material. Air is a better insulator than most common insulating materials, in addition, it is transparent. Using double glasses makes an Enclosure restricting the air in between, which is known as a *double-pane window*. Other examples of enclosures include wall *cavities*, *solar collectors*, and *cryogenic chambers* involving concentric cylinders or spheres. The fluid in the enclosure does not remain stationary due to heating making a rotationary motion within the enclosure that enhances heat transfer through the enclosure.



(a) Hot plate at the top



(b) Hot plate at the bottom

The heat transfer through a horizontal enclosure depend on whether the hotter plate is at the top or at the bottom. When the *hotter plate is at the top*, no convection currents will develop in the enclosure, since the lighter fluid will always be on top of the heavier fluid. Heat transfer in this case will be by *pure conduction*, and we will have Nu=1. When the *hotter plate is at the bottom*, the heavier fluid will be on top of the lighter fluid, and there will be a tendency for the lighter fluid to topple the heavier fluid and rise to the top, where it will come in contact with the cooler plate and cool down. Until that happens, however, the heat transfer is still by pure conduction and Nu=1. When $Ra>1708$, the buoyant force overcomes the fluid resistance and initiates natural convection currents, which are observed to be in the form of hexagonal cells called *Bénard cells*. For $Ra>3\times10^5$, the cells break down and the fluid motion becomes turbulent.

The Rayleigh number for an enclosure is determined from

$$\mathrm{Ra}_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\mathrm{Pr} \tag{4.19}$$

where the L_c is the distance between the hot and cold surfaces, and T_1 and T_2 are the temperatures of the hot and cold surfaces. All fluid properties are estimated at the average fluid temperature

$$T_{ave}=(T_1+T_2)/2 \tag{4.20}$$

4.5.1 Effective Thermal Conductivity

When the Nusselt number is known, the rate of heat transfer through the enclosure can be determined from

$$\dot{Q} = hA_s(T_1 - T_2) = k\mathrm{Nu}A_s\frac{T_1 - T_2}{L_c} \tag{4.21a}$$

$$\dot{Q}_{cond} = kA_s \frac{T_1 - T_2}{L_c} \qquad (4.21b)$$

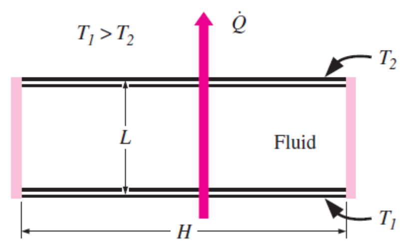
The convection heat transfer in an enclosure is analogous to heat conduction across the fluid layer in the enclosure. It provided that the thermal conductivity k is replaced by *kNu*, [*The fluid in an enclosure behaves like a fluid whose thermal conductivity is kNu as a result of convection currents*]. Therefore, the quantity *kNu* is called the *effective thermal conductivity* of the enclosure. That is,

$$k_{eff} = k\text{Nu}$$

Note that for the special case of Nu=1, the *effective thermal conductivity* of the enclosure becomes equal to the *conductivity of the fluid*. This is expected since this case corresponds to pure conduction.

4.5.2 Horizontal Rectangular Enclosures

The following correlations are sued for horizontal enclosures that contain air:



$$\text{Nu} = 0.195\text{Ra}_L^{1/4} \qquad 10^4 < \text{Ra}_L < 4 \times 10^5$$
$$\text{Nu} = 0.068\text{Ra}_L^{1/3} \qquad 4 \times 10^5 < \text{Ra}_L < 10^7 \qquad (4.22)$$

These relations can also be used for other gases with 0.5 < *Pr* < 2.

The correlation for horizontal enclosures heated from below using water, silicone oil, and mercury is;

$$\text{Nu} = 0.069\text{Ra}_L^{1/3}\,\text{Pr}^{0.074} \qquad 3 \times 10^5 < \text{Ra}_L < 7 \times 10^9 \qquad (4.23)$$

$$Nu = 1 + 1.44\left[1 - \frac{1708}{Ra_L}\right]^+ + \left[\frac{Ra_L^{1/3}}{18} - 1\right]^+ \qquad Ra_L < 10^8 \tag{4.24}$$

The notation [ ]$^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero.

4.5.3 Inclined Rectangular Enclosures

Heat transfer through an inclined enclosure depends on the aspect ratio H/L and the tilt angle θ from the horizontal.

$$Nu = 1 + 1.44\left[1 - \frac{1708}{Ra_L \cos\theta}\right]^+ \left(1 - \frac{1708(\sin 1.8\theta)^{1.6}}{Ra_L \cos\theta}\right) + \left[\frac{(Ra_L \cos\theta)^{1/3}}{18} - 1\right]^+ \tag{4.25}$$

for $Ra_L < 10^5$, $0 < \theta < 70°$, and $H/L \geq 12$

The notation [ ]$^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero. Note that this relation reduces to Eq. of horizontal enclosures for θ=0°.

For enclosures with smaller aspect ratios (H/L<12), the next correlation can be used provided that the tilt angle is less than the critical value θ_{cr} listed in Table 9–2.

TABLE 9–2

Critical angles for inclined rectangular enclosures

Aspect ratio, H/L	Critical angle, θ_{cr}
1	25°
3	53°
6	60°
12	67°
> 12	70°

$$Nu = Nu_{\theta=0°}\left(\frac{Nu_{\theta=90°}}{Nu_{\theta=0°}}\right)^{\theta/\theta cr}(\sin\theta_{cr})^{\theta/(4\theta_{cr})}, \qquad 0° < \theta < \theta_{cr} \text{ Use Table 9-2}$$

$$Nu = Nu_{\theta=90°}(\sin\theta)^{1/4}, \qquad \theta_{cr} < \theta < 90°, \text{ any } H/L \tag{4.26}$$

$$Nu = 1 + (Nu_{\theta=90°} - 1)\sin\theta, \qquad 90° < \theta < 180°, \text{ any } H/L$$

4.5.4 Vertical Rectangular Enclosures

$$Nu = 0.18\left(\frac{Pr}{0.2+Pr}Ra_L\right)^{0.29} \qquad \begin{matrix} 1 < H/L < 2 \\ \text{any Prandtl number} \\ Ra_L\,Pr/(0.2+Pr) > 10^3 \end{matrix} \qquad (4.27)$$

$$Nu = 0.22\left(\frac{Pr}{0.2+Pr}Ra_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \qquad \begin{matrix} 2 < H/L < 10 \\ \text{any Prandtl number} \\ Ra_L < 10^{10} \end{matrix} \qquad (4.28)$$

For vertical enclosures with larger aspect ratios,

$$Nu = 0.42\,Ra_L^{1/4}\,Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3} \qquad \begin{matrix} 10 < H/L < 40 \\ 1 < Pr < 2\times10^4 \\ 10^4 < Ra_L < 10^7 \end{matrix} \qquad (4.29)$$

$$Nu = 0.46Ra_L^{1/3} \qquad \begin{matrix} 1 < H/L < 40 \\ 1 < Pr < 20 \\ 10^6 < Ra_L < 10^9 \end{matrix} \qquad (4.30)$$

4.6 Combined Natural and Forced Convection

The natural convection heat transfer is usually ignored when it involves forced convection. The error involved in ignoring natural convection is negligible at high velocities but may be considerable at low velocities. Therefore, it is desirable to have a criterion to assess the relative magnitude of natural convection in the presence of forced convection.

It is observed that the parameter (Gr/Re^2) represents the ratio of the natural convection relative to the forced convection because the convection heat transfer coefficient is a strong function of the *Re* in forced convection and the Gr in natural convection.

From the Figure, it is noted that natural convection is negligible when $Gr/Re^2 < 0.1$, forced convection is negligible when $Gr/Re^2 > 10$, and neither is negligible when $0.1 < Gr/Re^2 < 10$ (*mixed convection*). Therefore, both natural and forced convection must be considered in heat transfer calculations when the Gr and Re^2 are 10 times the other. Note that forced convection is small relative to natural convection only in the rare case of extremely low forced flow velocities.

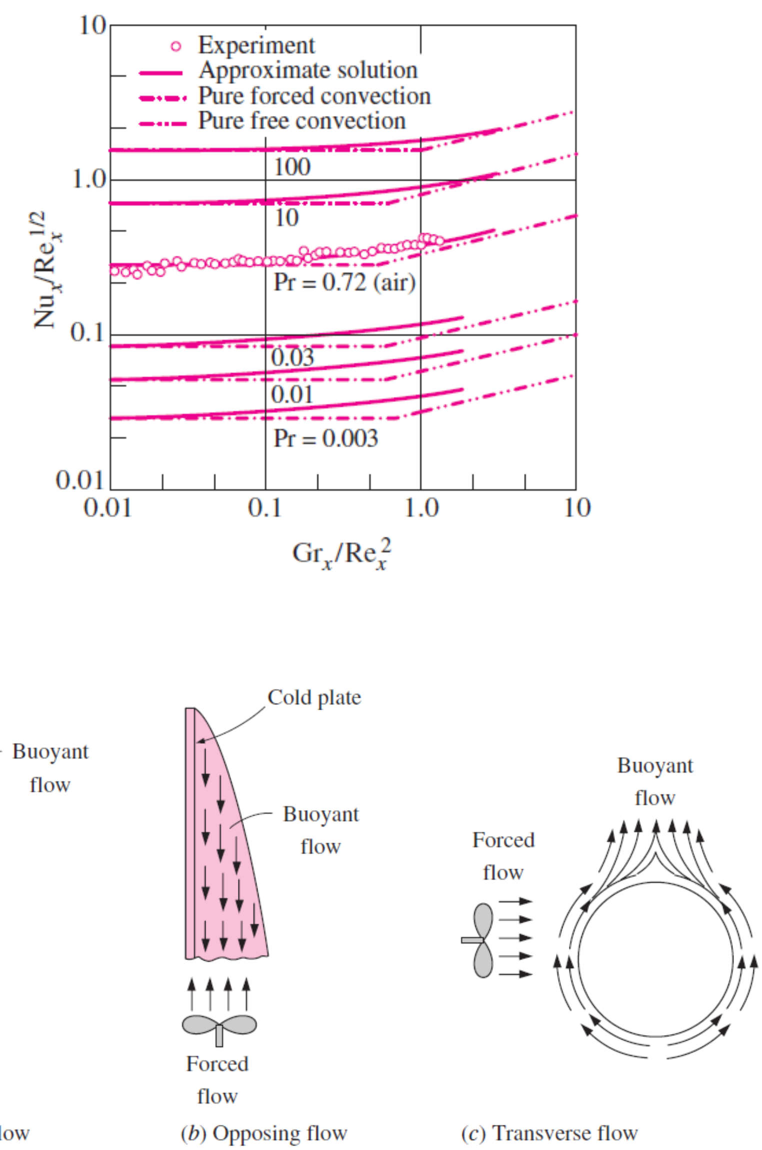


FIGURE 9–33

Natural convection can *enhance* or *inhibit* heat transfer, depending on the relative directions of *buoyancy-induced motion* and the *forced convection motion.*

1. *Assisting flow*: natural convection assists forced convection and enhances heat transfer.

2. *Opposing flow*: natural convection resists forced convection and decreases heat transfer.

3. *Transverse flow*; enhances fluid mixing and thus enhances heat transfer.

$$Nu_{combined} = (Nu^n_{forced} \pm Nu^n_{natural})^{1/n} \qquad (3.31)$$

where Nu_{forced} and $Nu_{natural}$ are determined from the correlations for *pure forced* and *pure natural* convection, respectively, each alone. The plus sign is for assisting and transverse flows and the minus sign is for opposing flows. The value of the exponent n varies between 3 and 4, depending on the geometry involved. Generally, $n = 3$ for vertical surfaces, while $n = 4$ for horizontal surfaces.